Computational Complexity in Three Areas of Computational Social Choice: Possible Winners, Unidirectional Covering Sets, and Judgment Aggregation

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ABSTRACT

This thesis studies the computational complexity of different problems from three areas of social choice. The first one is voting, and especially the problem of determining whether a distinguished candidate can be a winner in an election with some kind of incomplete information. The second setting is in the broader sense related to the problem of determining winners. Here the computational complexity of problems related to minimal upward and downward covering sets are studied. The last area is judgment aggregation, where judges have to report their judgments over a set of possibly interconnected propositions and a collective judgment set is determined by some aggregation procedure. In contrast to the problems mentioned above we do not study the complexity of some kind of "winner"-problem, but the complexity of two forms of influencing the outcome, namely manipulation and bribery.

1. INTRODUCTION

Computational social choice is at the interface between social choice theory and computer science, with a bidirectional transfer between these two disciplines. We focus on the study of the computational complexity of problems coming from social choice theory.

One central problem in social choice is that of winner determination in elections. From a computational point of view it is desirable that the winner can be determined in polynomial time. Associated with this problem is the possible winner problem. Here the question is whether an election, which is in some sense incomplete, can be completed such that a distinguished candidate wins the election. In contrast to the winner problem, it is not always desirable that possible winners can be computed in polynomial time, since it may give incentive to some kind of manipulation in the voting process. The first part of the thesis deals with several different possible winner problems.

Also related to the winner problem in voting are solution concepts for dominance graphs, as they may result from a pairwise majority relation. A solution concept is a way of identifying the "most desirable" elements of such dominance graphs. We study the complexity of various problems related to so-called upward and downward covering sets in the second part of this thesis. We show hardness and completeness not only for P and NP, but also for coNP and Θ_2^p, and we show membership in Σ_2^p.

The last part of this thesis is concerned with judgment aggregation. Here the task is not do determine a winner, but to aggregate the individual judgment sets over possibly interconnected propositions. We study manipulation and bribery in such judgment aggregation processes, the former asks the question whether a judge has an incentive to report an untruthful judgment set, and in the latter an external actor seeks to change the outcome by bribing some of the judges. Again, this may be seen as undesirable, hence showing NP-hardness can be seen as providing some kind of protection against manipulation and bribery, since then it cannot be determined in polynomial time (unless P equals NP) whether there is a successful manipulation or bribery action. In addition to classical complexity results, we also study the parameterized complexity of these problems.

2. POSSIBLE WINNER

The POSSIBLE WINNER problem was first defined by Konczak and Lang [10]. Given an election $E = (C, V)$ with the set of candidates C and a list of ballots V which are partial orders over the set of candidates, and a distinguished candidate $p \in C$, the POSSIBLE WINNER problem asks if it is possible to complete the votes in V such that p wins the election. This problem was studied for the class of pure scoring rules by Betzler and Dorn [6]. Their result was one step away from a full dichotomy since the complexity for one specific scoring rule was left open. We prove that the missing case is also NP-complete and so complete a dichotomy result for the important class of pure scoring rules [5].

In the original POSSIBLE WINNER problem there is no restriction on the structure of the ballots. One variant of this problem is POSSIBLE WINNER WITH RESPECT TO THE ADDITION OF NEW CANDIDATES. Here the votes are partial in the sense that the same set of candidates does not occur in all votes. Obviously this problem is a special case of POSSIBLE WINNER, hence polynomial time algorithms carry over to this special case. As the name suggests the intention to study this problem is that after the ballots have been cast there are new candidates entering the election (suppose for example that a new time slot for a meeting becomes available after the preferences have already been cast). We show amongst other things that POSSIBLE WINNER WITH RESPECT TO THE ADDITION OF NEW CANDIDATES is NP-

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complete for one class of pure scoring rules if one candidate is added [4].

Another source of uncertainty in an election may be that the voting rule used to aggregate the ballots is unknown. We show that the problem POSSIBLE WINNER UNDER UNCERTAIN VOTING SYSTEM is NP-complete if the voting rule is chosen from a subclass of the scoring rules [4]. Furthermore we show that this problem is polynomial time solvable for the family of Copeland$^\alpha$ elections, where α is a rational number in $[0, 1]$.

One further restriction on the form of the ballots are top and/or bottom truncated ballots. If the set of alternatives is too large one might ask the voters to specify only a ranking of their top and/or bottom candidates. POSSIBLE WINNER WITH TOP/BOTTOM-TRUNCATED BALLOTS asks if there is an extension of those ballots into complete ones such that the distinguished candidate wins. We study the computational complexity of these problems [3]. Again, they are special cases of POSSIBLE WINNER, hence the polynomial time algorithms carry over. We prove that POSSIBLE WINNER WITH TOP/BOTTOM-TRUNCATED BALLOTS can be solved in polynomial time for k-approval, whereas Betzler and Dorn [6] showed that POSSIBLE WINNER for k-approval is NP-complete for all values except 1 and $m - 1$ if there are m candidates.

3. UNIDIRECTIONAL COVERING

Minimal upward and downward covering sets are subsets of the alternatives of a dominance graph which satisfy certain notions of external and internal stability. The complexity of minimal upward and minimal downward covering sets was first studied by Brandt and Fischer [7], where NP-hardness was shown for the problem of deciding whether an alternative is contained in some minimal upward or downward covering set. We extend the complexity-theoretic study of problems related to upward and downward covering sets [1]. By applying Wagner's technique (see [11]) we raise the NP-hardness lower bound to the Θ_2^p level of the polynomial hierarchy. Furthermore we consider minimum-size upward and downward covering sets and consider five different decision problems for all these solution concepts. We show completeness for NP, coNP, and Θ_2^p. Besides the decision problems we also study the problems of finding minimal and minimum-size upward and downward covering sets. Our main result is, that this is not possible in polynomial time, unless P equals NP.

4. JUDGMENT AGGREGATION

The aim of a judgment aggregation process is to aggregate the individual judgment sets of the judges over possibly interconnected propositions. The complexity-theoretic study was initiated by Endriss et al. [8, 9]. We pursue their direction and study the complexity of manipulation in judgment aggregation [2]. A judgment aggregation scenario is said to be manipulable if one judge has an incentive to report an untruthful judgment set as this yields a more favorable outcome for him, where the distance between two judgment sets is measured by the hamming distance. We show W[2]-hardness, and hence NP-hardness, for the class of uniform premise based quota rules and different natural parameters, but also membership in P for certain restrictions on the agenda. Besides the manipulation problem, we also investigate bribery in judgment aggregation. This work is inspired by different bribery problems in voting theory. W[2]-hardness is shown for several different bribery problems and different natural parameters for the premise based procedure, and again membership in P for restricted problem instances.

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