Posting Prices for a Perishable Item

(Extended Abstract)

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ABSTRACT

In this paper, we study the problem of posting take-it-or-leave-it prices for a perishable item to maximize the seller's expected revenue. Agents arrive independently over time, and each of them makes a purchase decision according to the price. We study the case that the seller has no prior information about agents' valuations, whereas the benchmark in the competitive analysis is an optimal mechanism that knows the value distribution. We propose a mechanism Γ_1 which obtains constant competitive ratios under various valuation models. We also conduct numerical simulations and validate the empirical performance of Γ_1.

CCS Concepts

•**Theory of computation** $\rightarrow$ **Computational pricing and auctions;** *Online algorithms;* •**Computing methodologies** $\rightarrow$ **Multi-agent systems;**

Keywords

online mechanism design; revenue maximization; competitive analysis; detail-free mechanism

1. INTRODUCTION

Imagine a scenario, where you intend to transfer a ticket for a given concert. You post the information on campus BBS and set a price for the ticket. Potential agents arrive online and each has a valuation for the ticket. As a senior user of the campus BBS, you are familiar with the visiting traffic and estimate it as a Poisson process. However, you have no information about how potential agents would value the ticket, except from a range of reasonable valuations. The price can be updated at any time, and the target is to maximize the expected revenue.

Posted-price mechanism in such a scenario combines multiple challenges: (i) Challenge of *detail-free* mechanism design [4], which does not use any information about the value distribution. (ii) Challenge of *online* mechanism design [5], which needs to deal with uncertainty about future arrivals. (iii) Challenge of *finite time* horizon, which constitutes a period of validity for the mechanism. In this work, we design posted-price mechanisms with revenue guarantees for selling a single item in a finite time period.

In a typical posted-price mechanism, the seller presents a take-it-or-leave-it price for each buyer, and the buyer makes a purchase decision with respect to that price. Posted-price mechanisms show many advantages over traditional auction-style mechanisms. It is shown that more and more *eBay* transactions are made by posting prices rather than auctions [2].

Detail-free mechanisms are desirable since having knowledge of value distributions is costly or unrealistic in many practical scenarios [1]. We evaluate the performance of such a mechanism by *competitive analysis* [3], which compares the revenue implemented by the mechanism (without any knowledge of value distributions) with the optimal one (with the knowledge of value distributions).

We study posted-price mechanism design in an online fashion, i.e. prices are posted immediately after every agent arrives without knowledge of future agents. This online condition reflects our real lives, because the decision problems in many buying and selling domains are inherently dynamic rather than static [6]. We assume that the seller is able to build an accurate model to predict the distribution of agents' arrivals, as it is not difficult for the seller to measure "footfall" of a shop or "visiting traffic" of a webpage.

In this work, we consider the item for sale is perishable, i.e. only available for a limited period. It is easy to think of examples in real life, such as a concert/movie ticket, or a time-limited coupon.

2. MODELS, MECHANISM, AND ANALYSIS

Suppose there is a seller who is interested in selling a single item, within a finite period T. At any time $t \in [0, T]$, the seller offers a take-it-or-leave-it price. Agents arrive online according to a Poisson process with known intensity λ.

Two valuation models are studied in this paper:

- **Identical Valuations:** Agents have a unified valuation v for the item, that is, all agents' valuations are the same.
- **Random Valuations:** Agents' valuations are independently drawn from a fixed distribution.

The valuations[1] of agents for the item are independently and identically drawn from a distribution function F. The seller does

[1]As the identical-valuation model can be seen as a special case of the random-valuation model, we formulate the problem in the language of the random-valuation model.

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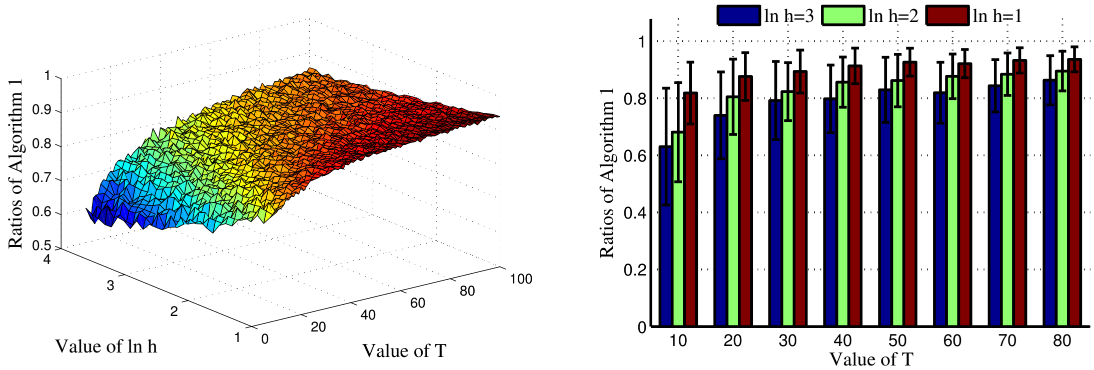

Figure 1: Uniform Distribution Samples

not know F, but knows that F belongs to the distribution family $\mathcal{F}$ with support $[v_{\min}, v_{\max}]$. $v_{\min}$ and $v_{\max}$ can be estimated by the seller. For simplicity, we normalize the range as $[1, h]$ for $h = v_{\max}/v_{\min} \geq 1$.

We use $p(t) : [0, T] \to \mathbb{R}_{\geq 0}$ to denote the *pricing strategy* of the seller. The agent who arrives at t, buys the item if and only if her valuation $v(t) \geq p(t)$, and in case she buys the item, she pays the price $p(t)$. Clearly, the posted-price mechanism is incentive compatible. Note that in the posted-price mechanism, the seller never learns the exact valuation of any arrived agent, but only observes the agent's decision: buy it or leave it.

First, we prove that the non-increasing pricing strategy (i.e., $p(t)$ is non-increasing with time t) is a dominant strategy for the seller. What's more, such a pricing strategy must maintain a bottom price (i.e., price 1) for the last period of the time horizon.

Second, we study the identical-valuation model. We present a deterministic posted-price mechanism Γ_1, which posts exponential decreasing prices (from the highest price h to the bottom price 1), and maintains the bottom price from time t_0 to deadline T. t_0 is a parameter to be optimized. In Γ_1, we define t_0 as the unique positive real root of the equation $\frac{\lambda t_0}{\ln h} = e^{\lambda(T-t_0)} - 1$. We show that Γ_1 has a competitive ratio of $\frac{1-e^{-\lambda(T-t_0)}}{1-e^{-\lambda T}}$.

Third, we study the random-valuation model. We assume value distributions satisfy the non-decreasing monotone hazard rate property, i.e., the hazard rate $H(x) = \frac{f(x)}{1-F(x)}$ is non-decreasing, where $F(x)$ is the cumulative distribution function and $f(x) = dF(x)/dx$ is the probability density function. We show that Γ_1 still achieves a good revenue compared to the benchmark. Specifically, Γ_1 has a competitive ratio of $\frac{1-\epsilon}{\kappa e}$, where $0 < \epsilon < 1$, and κ is a parameter depending on λT and h.

3. SIMULATION RESULTS

We conduct simple simulations to evaluate Γ_1 under illustrative value distributions and Poisson arrival models. We focus on examining the performance of Γ_1 on revenue compared with the *offline optimal revenue*. Fig.1 shows the simulation results of our mechanism under uniform distributions.

The empirical performances of Γ_1 is much better than the worst-case bounds (as indicated by competitive ratio). For example, when $T = 30, \lambda = 1$ and $\ln h = 2$, the empirical result of Γ_1 gives ratio 0.833, while the competitive analysis gives a ratio around 0.132. It can also be seen that the ratio of Γ_1 increases with T but decreases with h (the opposite for standard deviation), which consists with the worst case analysis since the competitive ratio is an increasing function of T and an decreasing function of h.

4. CONCLUSIONS

In this paper, we studied the online posted-price mechanisms for selling a single item in a finite time horizon. We designed a detail-free mechanism with revenue that is close to the optimal revenue. We proved in the identical-valuation model that Γ_1 achieves the optimal competitive ratio among all the deterministic posted-price mechanisms. In the random-valuation model, we showed that Γ_1 achieves constant competitive ratio if the valuation distributions are MHR (Monotone Hazard Rate).

Furthermore, we conjecture that $1/e$ is the tight upper bound for the competitive ratio in the random-valuation model. It remains open to study posted-price mechanism in conjunction with auctions and multiple item settings.

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