120 Million Agents Self-Organize into 6 Million Firms: A Model of the U.S. Private Sector

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ABSTRACT

An agent model is described at full-scale with the U.S. private sector, consisting of some 120 million agents. Using data on the population of U.S. firms the model is calibrated to closely reproduce firm sizes, ages, growth rates, job tenure and labor flows, along with several other empirically-important facts. It consists of a coalition formation model in which the Nash equilibria are dynamically unstable for sufficiently large coalitions. When agents are free to join coalitions where they are made better off there results a steady-state distribution of coalitions. The agent level is in perpetual disequilibrium but the aggregate level approaches a steady-state. This model represents a significant advance over conventional approaches to economic modeling, made possible by large-scale, parallel agent computing.

CCS Concepts

Computing methodologies: Parallel computing methodologies: massively parallel algorithms: self-organization; Artificial intelligence: distributed artificial intelligence; Modeling and simulation; simulation types and techniques, agent/discrete models, distributed simulation; **Applied computing**: Law, social and behavioral sciences: economics.

General Terms

Algorithms, Economics, Experimentation, Verification.

Keywords

Agent solutions of significant social and economic impact; coalition formation; behavioral game theory; organizations and institutions; Social simulation.

1. INTRODUCTION

Over the last decade the U.S. private sector workforce has ranged from 115 to 120 million employees, with nearly 3 million workers changing employers *each month* on average [1]. Over this same period there were, each year, 5.7-6.0 million firms with employees of which, on average, 100 thousand went out of business *monthly* while a comparable number started up [2]. Such high levels of turnover in the American economy—1 in 40 workers changing employers monthly, 1 in 60 firms terminating its operations—portrays a kind of *perpetual economic flux* in the U.S. private sector. How should we interpret such persistent adjustments and reorganizations of production? If we stipulate that the economy is in general equilibrium then there is no way to realize micro-dynamics except by the imposition of external shocks. Can microeconomic models *endogenously* produce the kinds of dynamics observed empirically when the incentives agents have to change jobs are fully represented?

Here I describe a microeconomic model capable of producing, *without* exogenous shocks, firm and labor dynamics of the size and type the U.S. economy experiences. While conventional explanations for these large labor flows exist [e.g., 3], here I provide a microeconomic explanation without the need for aggregate shocks. Also reproduced are a number of cross-sectional properties of U.S. businesses. Over the past decade there have appeared increasing amounts of micro-data on U.S. firms, including administratively *comprehensive* (tax record-based) data on firm sizes, ages, growth rates, labor productivity, job tenure, and wages. Extant theories place few restrictions on these data.[1] Lucas [10] derives Pareto-distributed firm sizes from a Pareto distribution of managerial talent. Luttmer [11, 12] obtains Zipf-distributed firm sizes and exponential firm ages [13] in a variety of general equilibrium settings, driven by exogenous shocks. Rossi-Hansberg and Wright [14] study establishment growth and exit rates arising in general equilibrium due to industry-specific productivity shocks. Elsby and Michaels [15] and Arkolakis [16] simulate heterogeneous firm growth rates due to productivity shocks. However, there are *many* more data on firm dynamics and labor flows to be explained. Here I develop a model that reproduces more than two dozen features of the empirical data *without* recourse to exogenous shocks—such shocks are not necessary in a model with worker-level dynamics.

The model draws together threads from various theoretical literatures. It is written at the level of individual agents and incentive problems of the type studied in the principal-agent literature manifest themselves. The agents work in perpetually novel environments, so contracts are incomplete and transaction costs are implicit. Each firm is a coalition of agents making the theory of coalition formation relevant [17]. Agent decisions generate firm growth and decline in the spirit of evolutionary economics [18]. Specifically, the model consists of a heterogeneous population of agents with preferences for income and leisure. Production takes place under increasing returns to scale, so agents who work together can produce more output per unit effort than by working alone. However, agents act non-cooperatively[2]: they select effort levels that improve their own

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[1] A generation ago Simon [4, 5] noted the inability of the neoclassical theory of the firm to explain empirical size distributions. Transaction cost [e.g., 6] and game theoretic explanations of the firm [e.g., 7, 8] make few empirical claims. Sutton [9] bounds the extent of intra-industry concentration, constraining the shape of size distributions.

[2] For a cooperative game theoretic view of firms see Ichiishi [19].

welfare, and may migrate between firms or start-up new firms when it is advantageous to do so. Analytically, Nash equilibria within a firm can be unstable. Large firms are ultimately unstable because each agent's compensation is imperfectly related to its effort level, making free-riding possible. Highly productive agents eventually leave large firms and such firms eventually decline. All firms have finite lives. The dynamics of firms perpetually forming, growing and perishing are studied. *This non-equilibrium regime provides greater welfare than equilibrium.*

These dynamics mean it is analytically difficult to relate agent level behavior to aggregate outcomes. Therefore, features that emerge at the firm population level are studied using agent-based computing [20-23]. When agent models are 'spun' forward in time macro-structure "grows" from the bottom-up [e.g., 24]. No equations governing the aggregate level are specified, nor do agents have either complete information or correct models for how the economy will unfold. Instead, they glean data inductively from the environment and from their social networks, through direct interactions, and make imperfect forecasts of economic opportunities. [25]. The macroscopic properties of the model *emerge* from the agent interactions. This methodology facilitates modeling agent heterogeneity [26], non-equilibrium dynamics [27], local interactions [28], and bounded rationality [29, 30]. The results in this paper should be of interest both to economists and multi-agent systems researchers working on coalition formation.

2. SINGLE FIRM ANALYTICS

Consider a group of agents A, $|A| = n$, engaged in team production, each agent contributing some amount of effort, generating team output.[3] Specifically, agent i has endowment $\omega_i > 0$ and contributes effort level $e_i \in [0, \omega_i]$, to the group. The total effort of the group is then $E = \Sigma_{i \in A} e_i$. The group produces output, O, as a function of E, according to $O(E) = aE + bE^\beta$, $\beta > 1$, without capital as in Hopenhayn [34].[4] For $b > 0$ there are increasing returns to effort.[5] Increasing returns in production means that agents working together can produce more than they can as individuals.[6] To see this, consider two agents having effort levels e_1 and e_2, with $\beta = 2$. As individuals they produce total output $O_1 + O_2 = a(e_1 + e_2) + b(e_1^2 + e_2^2)$, while working together they make $a(e_1 + e_2) + b(e_1 + e_2)^2$. Clearly this latter quantity is at least as large as the former since $(e_1 + e_2)^2 \geq e_1^2 + e_2^2$. Agents earn according to a compensation rule. For now consider agents sharing total output equally: at the end of each period all output is sold for unit price and each agent receives an O/N share of the total output.[7] Agents have Cobb-Douglas preferences for income and leisure, parameterized by θ. All time not spent working is spent in leisure, so agent i's utility can be written as a function of its effort, e_i, and the effort of other agents, $E_{\sim i} = E - e_i$ as

$$U_i(e_i; \theta_i, \omega_i, E_{\sim i}, n) = \left(\frac{O(e_i, E_{\sim i})}{n}\right)^{\theta_i} (\omega_i - e_i)^{1-\theta_i}$$

Consider the individual efforts of agents to be unobservable. From team output, O, each agent i determines E and, from its contribution to production, e_i, can figure out $E_{\sim i}$. Agent i then selects effort $e_i^*(\theta_i, \omega_i, E_{\sim i}, n) = \arg max\, U_i(e_i)$. For $\beta = 2$, this can be solved exactly, but as we will have occasion to use values not equal to 2 the solution is not written out here. It turns out that e^* does not depend on n but does depend on $E_{\sim i}$—the effort put in by the other agents. Optimal effort decreases monotonically as 'other agent effort,' $E_{\sim i}$, increases. For each θ_i there exists some $E_{\sim i}$ beyond which it is rational for agent i to put in no effort. For constant returns, e_i^* decreases linearly with $E_{\sim i}$ with slope $\theta_i - 1$. Furthermore, it has been shown in [41] that

Proposition 1**:** Nash equilibrium exists and is unique [42].

Proposition 2: There exists a set of efforts that Pareto dominate Nash equilibrium [43], a subset of which are Pareto optimal. These (a) involve larger effort levels than the Nash equilibrium, and (b) are not individually rational.

Proposition 3**:** Nash equilibria are dynamically unstable for sufficiently large group size.

3. MANY FIRMS: MAIN RESULTS

With these results in hand we can now consider a large population of agents in which many teams form simultaneously. If one or more of these teams becomes unstable some of the agents will look for employment in other teams, or perhaps they will form new teams if it makes them better off. What happens overall? Do lots of little teams form or a few big ones? Is a static equilibrium of agents in teams reached if we wait long enough? Are patterns produced in the population of teams that are recognizable vis-á-vis real firms? Here I show that such patterns *do* arise and closely resemble data on U.S. firms.

I study the formation of teams within a population using software agents. In the agent-based model total output of a firm consists of both constant and increasing returns. Preferences and endowments, θ and ω respectively, are heterogeneous across agents. When agent i acts it searches over $[0, \omega_i]$ for the effort maximizing its next period utility. Because many firms will arise in the computational model, it is necessary to specify how agents move between firms. Each agent has an exogenous social network, a random graph, consisting of ν_i other agents. It considers (a) staying in its current firm, (b) joining ν_i other firms—in essence an on-the-job search over its social network [44, 45]—and (c) starting up a new firm. It chooses the option that yields greatest utility. Since agents evaluate only a small number of firms their information is very limited. We utilize 120 million agents, roughly the size of the U.S. private sector. Specifically, about 5 million agents are activated each period, corresponding to one calendar month, in rough accord with job search frequency [46] and closely approximating the distribution of job tenure. The 'base case' parameterization of the model in table 1 was developed by seeking good fits to the many empirical data described subsequently.[8]

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[3] The model derives from Canning [31], Huberman and Glance [32] and Glance *et al.* [33].

[4] While $O(E)$ relates inputs to outputs, like a production function, E is not the choice of a single decision-maker, since it results from the actions of autonomous agents. Thus, $O(E)$ cannot be made the subject of a math program, as in production theory, yet describes production possibilities.

[5] Increasing returns goes back at least to Marshall [35] and was the basis of theoretical controversies in economics in the 1920s [36, 37]. Recent work on increasing returns is reprinted in Arthur [38] and Buchanan and Yoon [39]. Colander and Landreth [40] give a history of the idea.

[6] There are many ways to motivate increasing returns, including 'four hands problems': two people working together are able to perform a task that neither could do alone, like carrying a piano up a flight of stairs.

[7] The model yields roughly constant total output, so in a competitive market the price of output would be nearly constant. Since there are no fixed costs, agent shares sum to total cost, which equals total revenue. The shares can be thought of as either uniform wages in pure competition or profit shares in a partnership.

[8] For model attributes with random values, each agent or firm is given a realization when it is instantiated.

Table 1: 'Base case' configuration of the agent model

Model Attribute	Value
$\|A\|$	120,000,000
a	*uniform* on [0, 1/2]
b	*uniform* on [3/4, 5/4]
β	*uniform* on [3/2, 2]
θ	*uniform* on [0, 1]
ω	1
compensation	equal shares
v	*uniform* on [2,6]
activation	uniform (all active each period)
activation/period	4% of total agents (4,800,000)
one period	one month of calendar time
initial condition	all agents in singleton firms

The model's execution can be summarized in pseudo-code:

```
• INSTANTIATE and INITIALIZE time, agent, firm,
  and data objects;
• REPEAT:
  o FOR each agent, activate it with 4%
    probability:
    ▪ Compute e* and U(e*) in current firm;
    ▪ Compute e* and U(e*) for starting up a
      new firm;
    ▪ FOR each firm in the agent's social
      network:
      • Compute e* and U(e*);
    ▪ IF current firm is not best choice THEN
      leave:
      • IF start-up firm is best THEN form
        start-up;
      • IF another firm is best THEN join
        other firm;
  o FOR each firm:
    ▪ Sum agent inputs and do production;
    ▪ Distribute output;
  o COLLECT monthly, annual statistics;
  o INCREMENT time and reset periodic
    statistics;
```

Each worker is represented as an agent in this model, and both agents and firms are software objects. It is important to emphasize that this is *not* a numerical model: there are no (explicit) equations governing the aggregate level; the only equations present are for agent decisions. "Solving" an agent model means marching it forward in time to see what patterns emerge (cf. Axtell 2000).

Initially, agents work alone. As each is activated it discovers it can do better working with another agent to jointly produce output. Over time some teams expand as certain agents find it welfare-improving to join them, while other teams contract as their agents discover better opportunities elsewhere. New firms are started-up by agents who lack better opportunities. Overall, once an initial transient passes an approximately stationary macrostate emerges.[9] In this macro steady-state agents continue to adjust their efforts and change jobs, causing firms to evolve, and so there is no equilibrium at the agent level.

[9] Movies are available at `css.gmu.edu/~axtell/Rob/Research/Pages/Firms.html#6`.

The number of firms varies over time, due both to entry—agents leaving extant firms for start-ups—and the demise of failing firms. In the U.S. about 6 million firms have employees. Figure 1 shows the number of firms (blue) in the steady-state over 25 years (300 months), in good agreement with the data.

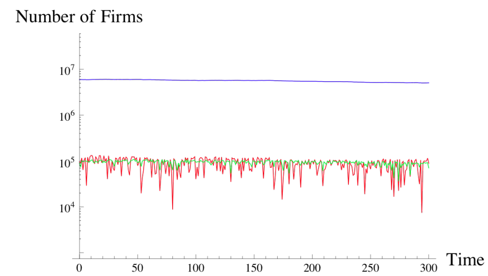

Figure 1: Typical time series for the total number of firms (blue), new firms (green), and exiting firms (red) over 25 years (300 months); note higher volatility in exits.

There are ~100K startups with employees in the U.S. monthly [2], quite close to the number produced by the model as shown in figure 6 (green). Exits are shown in red. The model predicts higher variability in firm exit than entry. Mean firm size in the U.S. is about 20 workers/firm [47]. Since there are 120 million agents in the model and the number of firms that emerges is approximately 6 million, mean firm size, as shown in figure 2, is very close to 20.

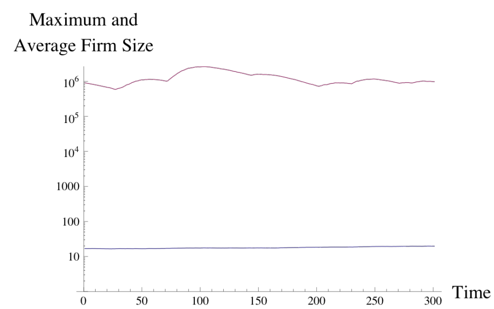

Figure 2: Typical time series for average firm size (blue) and maximum firm size (magenta)

Also shown in figure 2 is the size of largest firm (red), which fluctuates around a million. The largest firm in the U.S. (Wal-Mart) employs about 1.3 million today.

Agents who work together improve upon their singleton utility levels with reduced effort, as shown in figures 3 and 4. This is the *raison d'être* of firms

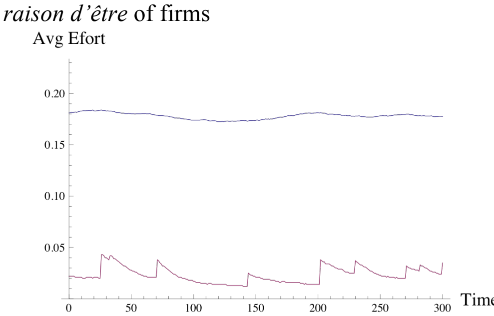

Figure 3: Typical time series for (a) average effort level in the population (blue) and in the largest firm (magenta)

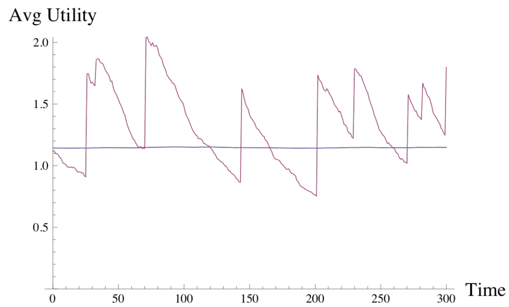

Figure 4: Typical time series for average utility (blue) and in the largest firm (magenta)

While efforts in large firms fluctuate, average effort overall is quite stable (figure 3). Much of the dynamism in the 'large firm' time series is due to the identity of the largest firm changing. Figure 4 shows the average agent utility (blue) is usually less than that in the largest firm (red). Occasionally utility in large firms falls below average, signaling that the large firm is in decline.

In the U.S. economy people change jobs with, what is to some, "astonishingly high" frequency [48: 1151]. Job-to-job switching (aka employer-to-employer flow) represents 30-40% of labor turnover, substantially higher than unemployment flows [46, 49-51]. Moving between jobs is intrinsic to this model. In figure 5 the level of monthly job changing at steady-state is shown (blue)—just over 3 million/month—along with measures of jobs created (red) and jobs destroyed (green). Job creation occurs in firms with net monthly hiring, while job destruction means firms lose workers (net). Job destruction is about 4x more volatile than job creation, comparable to U.S. data [52].

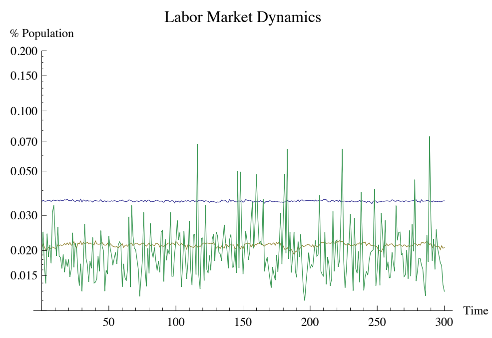

Figure 5: Typical monthly job-to-job changes (blue), job creation (yellow) and destruction (green)

Watching firms form, grow, and die in the model movies (see footnote 9), one readily sees the coexistence of big firms, medium-sized ones, and small ones. At any instant there exists a distribution of firm sizes in the model. At steady-state, sizes are skew, with a few big firms and larger numbers of progressively smaller ones. Typical model output is shown in figures 6 and 7 for firm size measured by employees and output. The modal firm size is 1 employee with the median between 3 and 4, in agreement with the data on U.S. firms. Firm sizes, *S*, are approximately Pareto distributed, the complementary *CDF* of which, $\bar{F}_S(s)$ is

$$Pr[S \ge s] \equiv \bar{F}_S(s;\, \alpha, s_0) = \left(\frac{s_0}{s}\right)^{\alpha}, s \ge s_0, \alpha \ge 0$$

where s_0 is the minimum size, unity for size measured by employees. The U.S. data are well fit by $\alpha \sim$ -1.06 [47], the line in figure 6.

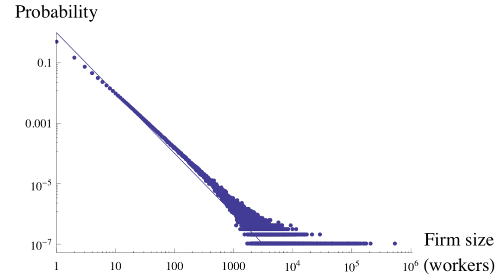

Figure 6: Stationary firm size distribution (*PMF*) by employees

The Pareto is a power law, and for $\alpha = 1$ is known as Zipf's law. Note that the power law fits almost the *entire distribution* of firm sizes. A variety of explanations for power laws have been proposed.[10] Common to these is the idea that such systems are far from (static) equilibrium at the microscopic (agent) level. Our model is non-equilibrium with agents regularly changing jobs.

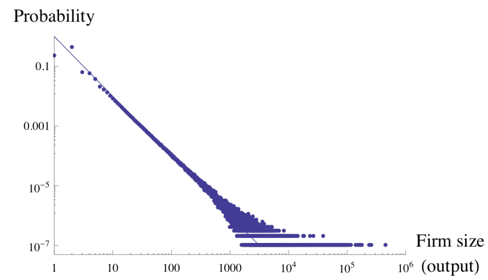

Figure 7: Stationary firm size distributions (*PMF*) by output

Firm output per employee is labor productivity. Figure 8 plots average firm output as a function of firm size. Fitting a line by several methods indicates that *ln*(*O*) scales linearly with *ln*(*S*) with slope very nearly 1.

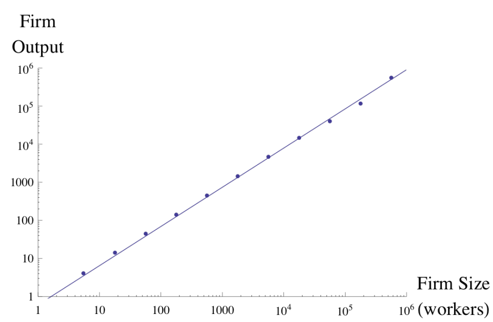

Figure 8: Constant returns at the aggregate level despite increasing returns at the micro-level

This represents essentially constant returns to scale, also a feature of U.S. output data; see Basu and Fernald [59]. That nearly *constant returns* occur at the *aggregate* level despite *increasing returns* at the *micro*-level suggests the difficulties of making inferences across levels. An explanation of why this occurs is apparent. High productivity firms grow by adding agents who work less hard than incumbents, thus such firms are driven toward the average productivity. In essence, each agent who changes jobs 'arbitrages' returns across firms.[11]

[10] Bak [53: 62-64], Marsili and Zhang [54], Gabaix, [55], Reed [56], and Saichev *et al*. [57]; for a review see [58].

[11] As output per worker represents wages in our model, there is little wage-size effect [60, 61].

It is well known that there is large heterogeneity in labor productivity across firms [e.g., 62]. Shown in figure 9 are data on all U.S. companies for three size classes: 1-99 employees (blue), 100-9,999 (red) and 10,000+ (green).

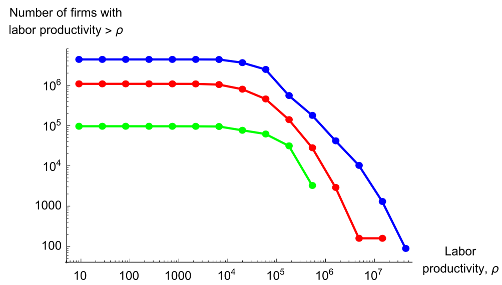

Figure 9: Labor productivity U.S. data (Census), in arbitrary units

Note the log-log coordinates, so the right tail is very nearly a power law with large slope. Souma *et al.* [63] have studied the productivity of Japanese firms and find similar results. Figure 10 is model output for the same size classes.

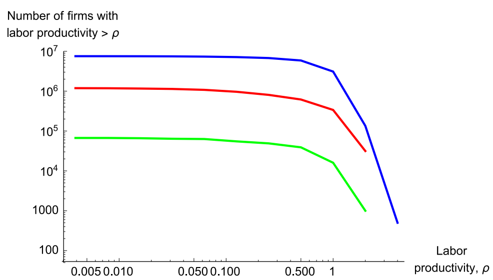

Figure 10: Labor productivity, model output in arbitrary units

Using data from the BLS Business Employment Dynamics program, figure 11 gives the age distribution (*PMF*) of U.S. firms, in semi-log coordinates, with each colored line representing the distribution reported in a recent year.

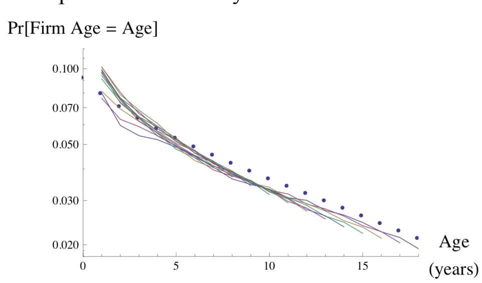

Figure 11: Firm age distributions (*PMF*s), U.S. data 2000-2011 (lines) and model output (points)

Model output is overlaid on the raw data as points and agrees reasonably well. Average firm lifetime and standard deviation are 14-15 years here. The curvature in the data implies that firm ages are better fit by the Weibull distribution than the exponential [64].

Data on U.S. firm ages is right censored so little systematic information is known about long-lived firms, except that they are rare [65]. Further, the role of mergers and acquisitions (M&A) makes the lifetime of a firm ambiguous, as when a younger firm buys an older one. This model can be run for a long time and makes strong predictions about the distribution of firm ages, along with the closely related idea of firm lifetimes, as shown in figures 12 and 13.

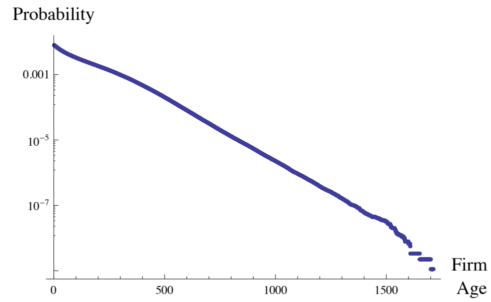

Figure 12: Firm age distribution (*PMF*s) in the long run (months)

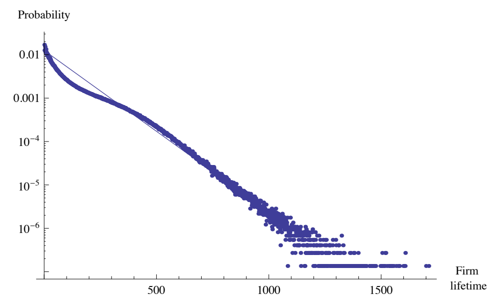

Figure 13: Firm lifetime distribution in the long run (months)

If firm ages were exactly exponentially distributed then the survival probability would be constant, independent of age [66]. Curvature in figure 11 indicates that survival probability depends on age. Empirically, survival probability *increases* with age [67-69]. This is shown in figure 15 for U.S. companies in recent years (lines) along with model output (points). The model slightly over-predicts the survival probabilities of young firms.

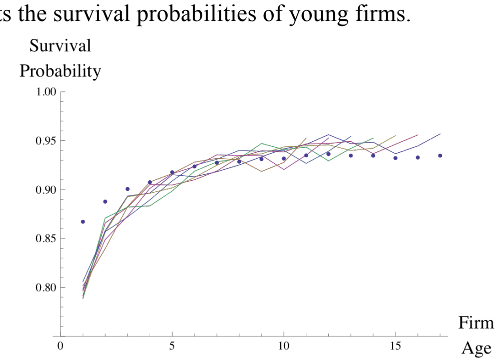

Figure 14: Firm survival probability increases with firm age and size, U.S. data 1994-2000 (lines) and model (points)

The joint distribution of size and age is shown in figure 15, a normalized histogram in *log* probabilities.

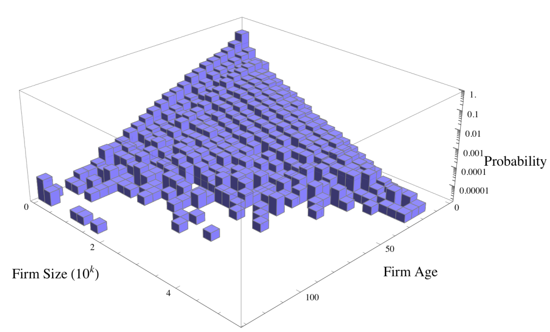

Figure 15: Histogram of the steady-state distribution of firms by *log*(S) and age in the model

Note that log probabilities decline approximately linearly as a function of age and $log(S)$. Many of the largest firms in the model are relatively young ones that grow rapidly, much like in the U.S. economy [e.g., 13, figure 1]

Call S_t a firm's size at time t. Its one period growth rate is $G \equiv S_{t+1}/S_t \in \boldsymbol{R}_+$.[12] In a population of firms consider G to be a stationary random variable. Gibrat's law of proportional growth [70] implies that if all firms have the same G then $S_{t+1} = GS_t$ is lognormally distributed at any t while the mean and variance of S grows with time [71: 40], i.e., S is not stationary. Adding firm birth and death processes can lead to stationary firm size distributions [72]. Historically, knowledge of G was limited by the relatively small samples of firm data available [e.g., 73]. Beginning with Stanley *et al*. [74], who analyzed data on publicly-traded U.S. manufacturing firms (Compustat), there has emerged a consensus that $g \equiv ln(G) \in \boldsymbol{R}$ is well-fit by the Subbotin or exponential power distribution.[13] This distribution embeds the Gaussian and Laplace distributions and has *PDF*

$$\frac{\eta}{2\sigma_g \Gamma(1/\eta)} exp\left[-\left(\frac{|g-\bar{g}|}{\sigma_g}\right)^{\eta}\right]$$

where $\bar{g}$ is the average log growth rate, σ_g is proportional to the standard deviation, and η is a parameter; $\eta = 2$ corresponds to the normal distribution, $\eta = 1$ the Laplace or double exponential.[14]

Data on g for all U.S. establishments has been analyzed by Perline *et al*. [82], shown as a histogram in figure 16 for 1998-99, decomposed into seven logarithmic size classes. Note the vertical axis is ln(frequency). In comparison to later years, e.g., 1999-2000, 2000-2001, this distribution is very nearly stationary.

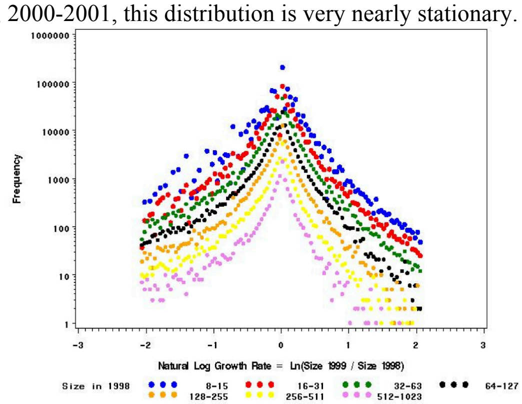

Figure 16: Histogram of annual g for U.S. establishments, by size

Perline *et al*. [82] find that $\eta \sim 0.60$ for the size 32-63 size class, lesser for smaller firms, larger for bigger ones. The gross statistical features of g are:

A. Growth rates *depend* on firm size—small and large firms have different g. This means that *Gibrat's law is false*: all firms do *not* have the same G.

B. Mode of $g \sim 0$, so mode(G) ~ 1, i.e., many firms do not grow.

C. There is more variance for firm decline ($g < 0$) than for growth ($g > 0$), i.e., there is more variability in job destruction than job creation [52], requiring an asymmetric Subbotin distribution [82].

D. Growth rate variance falls with firm size [67, 68, 74, 83, 84].

There are at least five other well-known regularities concerning firm growth rates that are *not* illustrated by the previous figure:

E. Mean growth is approximately 0;

F. Mean grow rate declines with firm size, and is positive for small firms, negative for large firms [52, 68, 84-87].

G. Mean growth *declines* with age [86, 88].

H. Mean growth *rises* with size, controlling for age [69].

I. Growth rate variance *declines* with firm age [86].

With these empirical features of firm growth rates as background, figure 17 shows distributions of g produced by the model for seven sizes of firrms, from small (blue) to large (purple) ones.

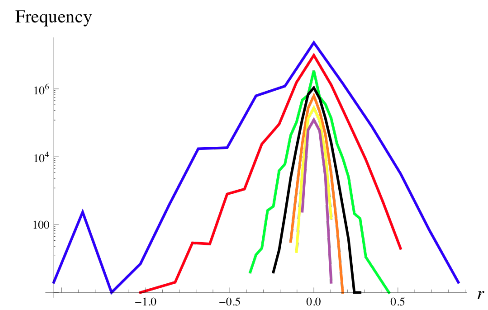

Figure 17: Distribution of annual g by firm size: 8-15 (blue), 16-31 (red), 32-63 (green), 64-127 (black), 128-255 (orange), 256-511 (yellow), and 512-1023 (purple)

In this plot we can see at least half of the empirical properties of firm growth: g clearly depends on firm size (*A*), with mode(g) = 0 (*B*) and $\bar{g} \sim 0.0$ (*E*). It is harder to see that there is more variance in firm decline than growth (*C*) but it is the case numerically. Clearly, variance declines with firm size (*D*). Figures 18 and 19 show mean growth rates as a function of firm size and age.

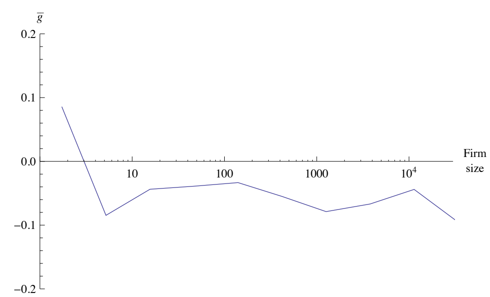

Figure 18: Dependence $\bar{g}$ on firm size, model output

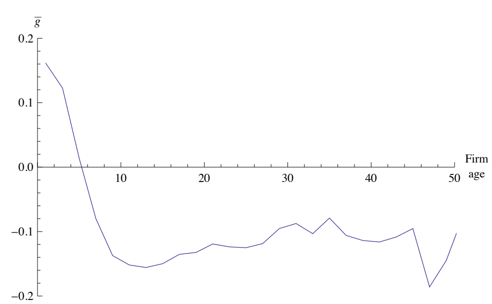

Figure 19: Dependence $\bar{g}$ on firm age, model output

[12] An alternative definition of G is $2(S_{t+1} - S_t)/(S_t + S_{t+1})$, making $G \in [-2, 2]$ [52]. Although advantageous because it keeps exiting and entering firms in datasets for one additional period, it obscures differences in growth rate tails by artificially truncating them.

[13] Subsequent work includes European pharmaceuticals [75] and Italian and French manufacturers [76, 77]. Bottazzi and Secchi [78] give theoretical reasons why g should have $\eta \sim 1$, having to do with the central limit theorem for the number of summands geometrically distributed [79]. Schwarzkopf [80, 81] argues that g is Levy-stable.

[14] For g Laplace-distributed, G follows the log-Laplace distribution, a kind of double-sided Pareto distribution [56], a combination of the power function distribution on (0, 1) and the Pareto on (1, ∞).

It is clear from thee figures that $\bar{g}$ declines with size (*F*) and similarly for age (*G*). For more than 30 years, since the work of Birch [85, 89], economists have debated the meaning of figures like 19. Specifically, given that small firms are often young and young firms small, it is not clear whether size or age plays the larger role in determining positive growth rates. Haltiwanger and co-workers [69, 88, 90] control for age and argue that it is not small firms that create jobs but rather young ones. The problem with such 'controls' for non-montonic relationships is that they mix effects across distinct (size, age) classes. The only actual way to understand the distinct effects of size and age is to show how they each effect $\bar{g}$. This is done in figure 20, where each firm is placed into a (size, age) bin and the average g computed locally.

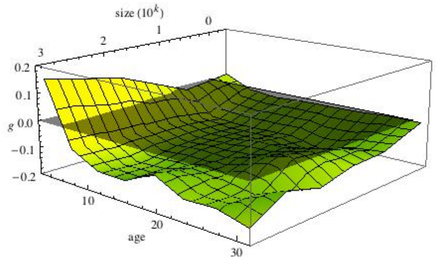

Figure 20: Dependence of $\bar{g}$ on firm size and age

To see precisely whether size or age matters most, a no growth ($\bar{g}$ = 0) plane is shown on the model's $\bar{g}$(size, age), revealing young firms grow the most with minor contribution from small firms.

Firm growth rate variability falls with size (*D*) and age (*I*). Figures 21 and 22 show this unconditionally for the model.

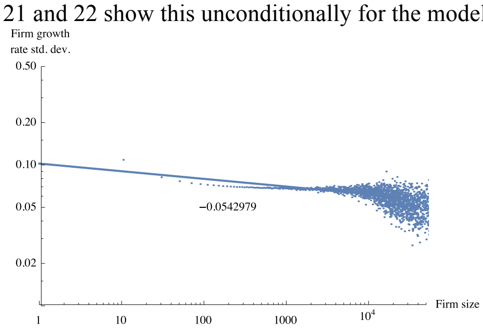

Figure 21: Dependence of the standard deviation of g on firm size

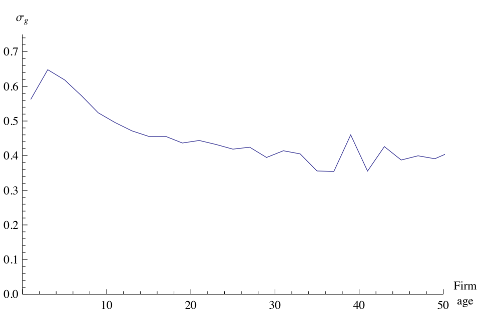

Figure 22: Dependence of the standard deviation of g on firm age

Specifically, the standard deviation of g falls with size in figure 21. Based on central limit arguments one expects this to be proportional to $S^{-\kappa}$, κ = ½ meaning the fluctuations are independent while κ < ½ implies they are correlated. Stanley *et al*. [74] find $\kappa \sim 0.16 \pm 0.03$ for publicly-traded firms (Compustat data) while Perline *et al*. [82] estimate $\kappa \sim 0.06$ for all U.S. establishments. From the model output $\kappa = 0.054 \pm 0.010$. A variety of explanations for $0 \leq \kappa \leq$ ½ have been proposed [91-96], all involving firms having more or less elaborate internal structure. Note that no internal structure exists in the present model, where firms are simply collections of agents, yet dependence of the standard deviation of g on size is present nonetheless.

At all times some firms are growing and others are declining. However, growing firms shed workers and declining ones hire. Figure 23 shows that growing firms hire in excess of their separation rate, while declining firms keep hiring even when separations are the norm, much like in the empirical data [1]. The 'hiring' line is quite comparable to the empirical result, but the 'separations' line is different—too few separations in the model.

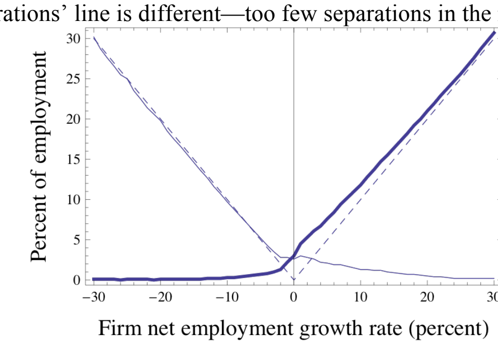

Figure 23: Model labor transitions as a function of firm growth

Having explored firms cross-sectionally, we next turn to the population of agents. Steady-state worker behavior is quantified here. While each agent's situation adjusts uniquely, at the population level there emerge robust statistical features.

While income and wealth are famously heavy-tailed [97, 98], *wages* are less so. A recent empirical examination of U.S. adjusted gross incomes argues that an exponential distribution fits the data below about $125K, while a power law better fits the upper tail [99]. Figure 24 is the model's income distribution.

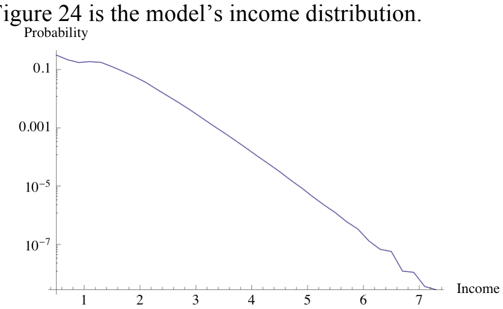

Figure 24: Wage distribution (arbitrary units)

Since incomes are nearly linear in this semi-log coordinate system, they are approximately exponentially-distributed.

Job tenure in the U.S. has a median of just over 4 years and a mean of about 8.5 years. The complementary-cumulative distribution for 2010 is figure 25 (points) with the straight line being the model output. As with income, these data are approximately exponentially distributed.

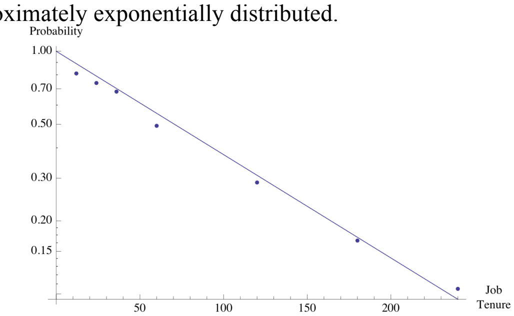

Figure 25: Job tenure (months) is exponentially-distributed in the U.S. (dots, binned) and in the model (line)

The base case of the model is calibrated to make these distributions coincide. That is, the number of agent activations per period is specified in order to make the line go through the points, thus defining the meaning of one unit of time in the model—a month. The many output of the model having to do with time—e.g., firm growth rates, ages—derive from this calibration.

In the model, as in the real world, workers regularly move between jobs. Here the *structure* of such migrations is studied, using a graph theoretic representation of inter-firm labor flows. Let each firm be a node (vertex) in such a graph, and an edge exists between two firms if a worker has migrated between the firms. Elsewhere this has been called the *labor flow network* [100]. In figure 26 a property of this network for the base case of the model is shown, the degree distribution, This closely reproduce data from Finland and Mexico [100], shown inset.

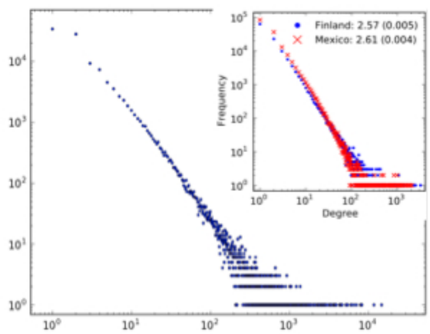

Fig 26: Degree distribution of the labor flow network

Each time an agent is activated it seeks higher utility, which is bounded from below by the singleton utility. Therefore, it must be the case that all agents prefer the non-equilibrium state to one in which each is working alone—the state of all firms being size one is Pareto-dominated by the dynamical configurations above. To analyze welfare of agents, consider homogeneous groups of maximum stable size, having utility levels shown in figure 3b, replotted in figure 27.

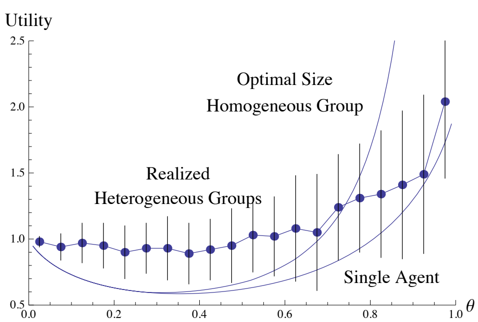

Figure 27: Utility in single agent firms, optimal homogeneous firms, and realized firms, by θ

Overall, this large-scale multi-agent model, calibrated to data using approximately a dozen parameters (table 1), has been shown here to closely reproduce nearly two dozen empirical facts concerning the U.S. economy, and represents a new approach to modeling industrial organization

4. REALIZING 120 MILLION AGENTS

Creating a model at full-scale with the U.S. economy has proved challenging in many dimensions. First there was the problem of *instantiating* so many agents. Hardware with large, flat (shared) memory spaces proved the most useful as all attempts to store this large agent population to disk was prohibitively slow. Second was the problem of fast execution. Many HPC architectures turns out to not be useful, such as cloud computing, running a single instance of the model on multiple machines, vector supercomputers and even GPUs. While each provided certain advantages, the dense interactions between agents in this model make inter-process communication voluminous and, because it is slow, prohibitive. The synchronous nature of vector supercomputers and GPUs led to problems by generating computational artifacts that had no meaningful interpretation. Instead of any of these technologies, a single dedicated workstation, with 32 cores and 256 GB of RAM, proved the most successful architecture. Using optimized C/C++ code and various threading libraries, for wide values of parameters the model reaches a near steady-state condition from the initial conditions of table 1 in approximately half a day a day of wall time. Given the expense of these model runs, the parameter optimization that led to the specification of table 1 was obtained heuristically.

5. SUMMARY AND CONCLUSIONS

Using the combination of a universe of micro-data on firms with large-scale multi-agent computing, a new kind of economic model has been created, its gross features being described above. It represents a novel contribution to both computational economics and multi-agent computer science. In terms of the former, most models in economics and finance today are solved for agent-level equilibria, while the model developed here involves agent-level disequilibria. This *feature* of the model permits us to generate endogenous economic dynamics that, as has been shown above, closely reproduce many economic facts. From a computer science point-of-view, the model shows new ways of building realistic, empirically-grounded coalition formation models, and demonstrates that these can be realized at massive scale.

For many years multi-agent systems researchers have been deeply enamored of more or less conventional game theoretic ideas, all of which involve agent-level equilibria. At the same time, social scientists have been attracted to MAS precisely because it is a technology that permits relaxation of the highly unrealistic strictures that analytical solutions require (e.g., fixed preferences, static prices, uniform probability of interaction, etc.). With models such as this one I hope to demonstrate to both communities that there is much to be gained from trade in new ideas for how to build empirically-salient models of human systems using multi-agent systems computer science.

6. ACKNOWLEDGMENTS

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