Parameterized Complexity of Multi-winner Determination: More Effort Towards Fixed-Parameter Tractability

Extended Abstract

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ABSTRACT

We study the k-committee selection rules minimax approval, proportional approval, and Chamberlin-Courant's approval. It is known that Winner Determination for these rules is NP-hard. Moreover, the parameterized complexity of the problem has also been studied with respect to some natural parameters. However, there are still numerous parameterizations that have not been considered. We revisit the parameterized complexity of Winner Determination for these rules by considering several important single parameters, combined parameters, and structural parameters, aiming at detecting as many fixed-parameter tractability results as possible.

KEYWORDS

multi-winner voting; parameterized complexity; approval voting; tree-width; maximum matching

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1 INTRODUCTION

Multi-winner voting rules have received a considerable amount of attention recently due to its significant applications in many areas [1, 9, 10, 19]. Calculating the winning candidates (k-committee) is of particular importance for voting. Intractability of winner determination for a multi-winner rule precludes the applications of this rule in practice. Fortunately, many multi-winner rules such as STV, Bloc, k-Borda, admit polynomial-time algorithms to determine the winners [7]. However, there are also important multi-winner rules for which the Winner Determination problem is NP-hard. Among them are minimax approval voting (MAV) [4, 14], proportional approval voting (PAV) [2, 11, 18], and Chamberlin-Courant's approval voting (CCA) [5, 17]. In spite of the intractability of Winner Determination, PAV, MAV and CCA in fact satisfy many desirable axiomatic properties [1, 9, 11–13], which advances the study of many topics around these rules. In addition, even when a problem is shown to be NP-hard, there are still prominent approaches to handle the problem efficiently. For instance, if one compromises on the quality of the solution, one could resort to approximation or heuristic algorithms. If, however, one insists on seeking an optimal solution, then one of the most prominent tools is arguably the parameterized complexity, which, by introducing a proper parameter can make a hard problem tractable with respect to the selected parameter. We refer to Chapter 11 of [8] for a nice survey of parameterized complexity used in computational social choice. This paper is concerned with the parameterized complexity of Winner Determination for MAV, PAV, and CCA.

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2 PRELIMINARIES

An *election* is a tuple $E = (C, V)$ where C is the set of candidates and V the multiset of votes, each of which is defined as a nonempty subset of C. We say a vote v *approves* a candidate c if $c \in v$. Let k be a positive integer such that $k \leq |C|$. A *k-committee selection rule* (k-multi-winner rule) maps each election (C, V) to a subset $w \subseteq C$ such that $|w| = k$. The subset w is called a *k-committee*. In this paper, we study the following k-committee selection rules. We exchangeably use the terms "vote" and "voter".

MAV The MAV score of a committee w with respect to an election (C, V) is $\text{MAV}(V, w) = \max_{v \in V} (|v \setminus w| + |w \setminus v|)$. MAV selects a k-committee with minimum MAV score.

CCA A voter v is satisfied with a committee w if and only if at least one of v's approved candidates is included in w, i.e., $v \cap w \neq \emptyset$. The CCA score of w with respect to (C, V), denoted $\text{CCA}(V, w)$, is the number of voters satisfied by w. CCA selects a k-committee with maximum CCA score.

PAV The PAV score of a committee w with respect to (C, V) is

$$\text{PAV}(V, w) = \sum_{v \in V, v \cap w \neq \emptyset} (1 + \frac{1}{2} + \cdots + \frac{1}{|v \cap w|}).$$

PAV selects a k-committee with maximum PAV score.

Let $\tau \in \{\text{PAV, CCA, MAV}\}$. The decision version of the winner determination problem for τ is defined as follows.

> Winner Determination for τ (τ-WD)
> *Input*: An election $E = (C, V)$ and two positive integers $k \leq |C|$ and d.
> *Question*: Is there $w \subseteq C$ such that $|w| = k$ and $\text{MAV}(V, w) \leq d$ for $\tau = \text{MAV}$, and $\tau(V, w) \geq d$ for $\tau \in \{\text{PAV, CCA}\}$?

	single parameter						combined parameter					structural parameter	
	d	m	n	k	$\bar{k}$	$(\triangle_V, \triangle_C)$	$k, \triangle_C$	$k, \triangle_V$	$\bar{k}, \triangle_C$	$\bar{k}, \triangle_V$	$d, \triangle_V$	ω	α
MAV	FPT [15]	FPT	FPT [15]	W[2]-h [15]	**W[2]-h**	(≥ 2, ≥ 3): NP-h [14] others: **P**	**FPT**	**FPT**	**FPT**	**W[2]-h**	FPT	w.r.t. ω, k **FPT**	**FPT**
CCA	$\mathbf{2^{O(d)}}$	FPT	n^n [3] $\mathbf{2^{O(n)}}$	W[2]-h [3]	**W[1]-h**	(≥ 2, ≥ 3): NP-h [16] others: **P**	**FPT**	**W[1]-h**	**FPT**	**W[1]-h**	FPT	$\mathbf{4^{\omega}}$	$\mathbf{4^{\alpha}}$
PAV	?	FPT	**FPT**	W[1]-h [2]	**W[1]-h**	(≥ 2, ≥ 3): NP-h[2] (≥ 3, 2): ? others: **P**	w.r.t. k $\triangle_C$, $\triangle_V$ **FPT**	W[1]-h [2]	w.r.t. $\bar{k}$, $\triangle_C$, $\triangle_V$ **FPT**	**W[1]-h**	**FPT**	w.r.t. ω, k **FPT**	**FPT**

Table 1: Our results are in boldface. For FPT-results with single-exponential time algorithms, we give the running time in the table with the big $O^*()$ omitted. Entries marked with "?" mean that the corresponding results remain open. ω is the parameter tree-width and α is the size of maximum matching of the incident graph of the given election.

In the following, let m be the number of candidates, i.e., $m = |C|$, n the number of votes, i.e., $n = |V|$, $\bar{k} = m - k$, $\triangle_V$ the maximum number of candidates a voter approves, i.e., $\triangle_V = \max_{v \in V}\{|v|\}$, and $\triangle_C$ the maximum number of voters a candidate is approved, i.e., $\triangle_C = \max_{c \in C}\{|v \in V \mid c \in v|\}$.

3 OUR CONTRIBUTION

Single-parameters. It is easy to see that Winner Determination for PAV, MAV and CCA is fixed-parameter tractable (FPT) with respect to m. Misra, Nabeel and Singh [15] proved that MAV-WD is FPT with respect to d and n, but becomes W[2]-hard with respect to k. Betzler, Slinko and Uhlmann [3] proved that CCA-WD is FPT with respect to n, but turn out to be W[2]-hard with respect to k. Moreover, they considered a dual parameter $R = n - d$. They proved that CCA-WD is NP-hard even for $R = 0$, but presented an FPT-algorithm with respect to the combined parameter $k + R$. Aziz et al. [2] proved that PAV-WD is W[1]-hard with respect to k even if every voter approves at most two candidates. We first close some gaps and improve an FPT-algorithm. Concretely, we propose an FPT-algorithm for PAV-WD with respect to n. With respect to the parameter d, we show that CCA-WD is FPT by developing a single-exponential time algorithm. For the parameter n, the FPT-algorithm for CCA-WD studied in [3] runs in time $O^*(n^n)$. We significantly improve the result by proposing an FPT-algorithm running in time $O^*(2^{O(n)})$. Second, we study a natural parameter $\bar{k} = m - k$, i.e., the number of candidates that are not expected to be in the k-committee. With respect to this parameter, we prove that MAV-WD is W[2]-hard, and CCA-WD and PAV-WD are W[1]-hard. Third, based on previous results we achieve some dichotomy results with respect to the two natural parameters $\triangle_C$ and $\triangle_V$. It is known that PAV-WD, MAV-WD and CCA-WD are already NP-hard when $\triangle_C = 3$ and $\triangle_V = 2$ [2, 14, 16]. We prove that MAV-WD and CCA-WD become polynomial-time solvable if $\triangle_C \leq 2$ or $\triangle_V \leq 1$, and PAV-WD becomes polynomial-time solvable if $\triangle_C = 1$, or $\triangle_V = 1$, or $\triangle_C = \triangle_V = 2$.

Combined parameters. Obviously, if a problem is FPT with respect to a parameter p then it is FPT with respect to any combined parameter which can be bounded from below by a computable function of p. Therefore, for MAV and CCA, and combinations of two single parameters, it only makes sense to study the following ones: $k + \triangle_V$, $k + \triangle_C$, $\bar{k} + \triangle_V$, $\bar{k} + \triangle_C$. We establish many FPT results with respect to these combined parameters. Concretely, we obtain FPT results for MAV-WD and CCA-WD with respect to both $k + \triangle_C$ and $\bar{k} + \triangle_C$. However, we show that MAV-WD is W[2]-hard and CCA-WD is W[1]-hard with respect to $\bar{k} + \triangle_V$. With respect to $k + \triangle_V$, we develop an FPT-algorithm for MAV-WD but show that CCA-WD is W[1]-hard. Concerning PAV, the reduction by Aziz et al. [2] implies that PAV-WD is W[1]-hard with respect to $k + \triangle_V$. We show that the same result holds for the combined parameter $\bar{k} + \triangle_V$ too. We are not able to show the fixed-parameter tractability of PAV-WD with respect to the single-parameter d, but we show that combining d with $\triangle_V$ leads to an FPT result. Moreover, if we combine k, $\triangle_C$ and $\triangle_V$, or combine $\bar{k}$, $\triangle_C$ and $\triangle_V$ we also have FPT results for PAV-WD.

Structural parameters. So far the most studied structural parameters for multi-winner determination are based on various concepts of restricted domains, such as single-peaked or single-crossing domains (see, e.g., [3, 6, 20]). In this paper, we study some different structural parameters. Given an election $E = (C, V)$ we can construct a bipartite graph G_E, called the *incident graph* of E, with vertex set $C \cup V$. There is an edge between a candidate $c \in C$ and a vote $v \in V$ if and only if $c \in v$. We study the tree-width of G_E and the size of a maximum matching of G_E. We prove that CCA-WD is FPT with respect to the tree-width of G_E, and MAV-WD and PAV-WD are FPT if we combine the tree-width of G_E and the parameter k. With respect to the size of a maximum matching of G_E, we present FPT results for all three voting rules.

Our results are summarized in Table 1.

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