# Concurrent Game Structures for Temporal STIT Logic

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## ABSTRACT

The paper introduces a new semantics for temporal STIT logic (the logic of *seeing to it that*) based on concurrent game structures (CGSs), thereby strengthening the connection between temporal STIT and existing logics for MAS including coalition logic, alternating-time temporal logic and strategy logic whose language are usually interpreted over CGSs. Moreover, it provides a complexity result for a rich temporal STIT language interpreted over these structures. The language extends that of full computation tree logic (CTL*) by individual agency operators, allowing to express sentences of the form "agent $i$ sees to it that $\varphi$ is true, as a consequence of her choice".

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## 1 INTRODUCTION

STIT logic (the logic of *seeing to it that*) by Belnap et al. [5, 16, 17] is one of the most well-known formal theories of agency. It is the logic of sentences of the form "group $J$ sees to it that $\varphi$ is true", denoted by $[J]\varphi$, where a group $J$ is defined to be a set of agents. Following [20], one might use the terms 'group STIT logic' and 'individual STIT logic' to designate, respectively, the family of STIT languages that contain a group agency operator $[J]$ for every group $J$ and the family of STIT languages that only contain an individual agency operator $[\{i\}]$ for every agent $i$.

Two variants of STIT have been studied in the literature: 'atemporal STIT' and 'temporal STIT' (T-STIT). At the syntactic level, the former corresponds to the family of languages for expressing properties of individual and group agency with no temporal operators. Notable examples are the languages studied by [3, 14, 15, 20]. The latter corresponds to extensions of atemporal STIT languages by temporal operators for expressing properties of agency in connection with time such as the temporal operator 'next' of linear temporal logic LTL [6, 25][1] as well as future and past tense operators of basic tense logic [16, 18, 32]. At the semantic level, atemporal STIT abstracts away from the branching-time account of agency and only considers one-shot interaction. On the contrary, T-STIT focuses on repeated (possibly infinite) interactions and requires a formal semantics of branching-time. This corresponds to the game-theoretic distinction between games in normal form and games in extensive form.[2]

Although STIT theory has a solid philosophical basis, at the current stage, its applicability to multi-agent systems (MAS) is limited for at least two reasons.

First of all, existing semantics for STIT use notions such as moment, history and 'not necessarily discrete' time that are unfamiliar and unattractive to most logicians in artificial intelligence (AI). The structures with respect to which STIT languages are interpreted highly differ from the structures that are traditionally used in the area of logics for MAS including coalition logic (CL) [22], alternating-time temporal Logic (ATL) [2, 13] and strategy logic (SL) [21]. These logics are usually interpreted over concurrent game structures (CGSs). CGSs have been widely used in AI to model interaction between multiple agents. Moreover, their connections with alternative models of interaction in AI including alternating transition systems [2], reactive modules [28], effectivity functions [12] and models of propositional control [4] have been clarified.

Secondly, the computational properties of STIT theory including decidability and complexity are far less studied and understood than those of CL, ATL and SL. Therefore, its potential for applications remains unclear, compared to existing logics for MAS. Few properties of STIT are known and all of them are limited either to atemporal STIT languages or to restrictive temporal STIT languages whose only temporal operator is the next-time operator. For instance, it is known that the satisfiability problem is undecidable for temporal and atemporal group STIT with more than two agents [15] and NEXPTIME-complete for both the atemporal individual STIT language [3] and the temporal individual STIT language restricted to the next-time operator [25].

The aim of this paper is to overcome these two limitations of STIT theory (i) by introducing a new semantics for STIT based on CGSs, and (ii) by providing a complexity result for a rich temporal individual STIT language — including operators 'next' and 'until' of LTL — interpreted over CGSs. Differently from the original Belnap et al.'s semantics, our CGSs semantics for STIT assumes time to be discrete. This assumption is fundamental for proving our complexity result, as the techniques we use are based on automata and only apply to discrete branching-time structures.

The paper is organized as follows. In Section 2, we first recall Belnap et al's definition of *BT+AC* structures and define a variant of these structures with discrete time (discrete *BT+ACs*). Then, in Section 3, we introduce a temporal STIT language that extends the language of full computation tree logic CTL* [9, 11, 24] by agency operators. We define the interpretation of this language relative to discrete *BT+ACs*. In Section 4, we present a new semantics for STIT based on CGSs and interpret the language of Section 3 over this

[1] The main feature of the language studied by [6] is that the temporal operator 'next' and the agency operator are fused into a single operator. In the language studied by [25] they are kept separated.

[2] The relationship between the semantics for atemporal STIT and games in normal form has been explored, e.g., by [19, 26, 27].

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class of structures. In Section 5, we provide two results: the tree-model property for our temporal STIT logic interpreted over CGSs and an equivalence result relative to the two semantics based on CGSs and discrete *BT+AC*s. Since the logic on the whole language is undecidable, we define in Section 6 the temporal *individual* STIT fragment which is proved in Sections 7, 8 and 9 to be decidable. For that purpose, a dedicated semantics is defined in Section 7. We prove in Section 8 that this semantics is equivalent to the CGS semantics. Finally, in Section 9, an automaton is constructed that recognizes exactly the models in the dedicated semantics satisfying a given formula. In Section 10, we conclude.

At the end of this introduction, we would like to mention the related work by [8] who extend the language of ATL by 'strategic' STIT operators in order to express that "group $J$ performs a strategy that, whatever strategy is taken by the others, ensures that a certain property $\varphi$ holds".[3] There are substantial differences between their work and our work: (i) Broersen et al. interpret their STIT-extension of ATL over alternating transition systems (ATSs), while we interpret our temporal STIT logic over CGSs; (ii) they do not prove any equivalence result between the semantics based on ATSs and the semantics based on *BT+AC*s for their STIT-extension of ATL, while we prove equivalence between the semantics based on CGSs and the semantics based on discrete *BT+AC*s for our temporal STIT language; (iii) they do not provide any decidability or complexity result for their language or for some fragments of it, while we do it for our language.

## 2 *BT+AC*-BASED SEMANTICS FOR T-STIT

We now consider *BT+AC* structures which were first introduced in STIT theory by Belnap et al. [5, 16]. Such structures are based on full trees of branching time temporal logics augmented with group-relative relations. The structures we are presenting here differ from Belnap et al.'s original ones in two minor respects. First, we replace the function of choice with choice-equivalence classes, with a move that is usual in STIT logics and will have no bearings in what follows. Secondly, the truth values of atomic propositions are assumed to be moment-determinate in a way consistent with branching-time temporal logics such as CTL*, while Belnap et al. assume that they depend on the history passing through the moment.

We start with the following definition of tree, defined as a set of moments and a branching-time temporal relation over them.

*Definition 2.1.* A tree is a pair $T = (Mom, \prec)$, where:

- $Mom$ is a nonempty set of moments;
- $\prec$ is a binary relation on $Mom$ that is serial, irreflexive, transitive, left-linear[4] and rooted[5]. We let $\succ$ be the inverse relation of $\prec$.

The notion of a history is also crucial in such structures.

*Definition 2.2.* Histories are sets $h, h', \ldots$ of moments that are linearly ordered by $\prec$ and are maximal for inclusion. $H_T$ is the set of all histories in the tree $T$, and $H_m$ is the set of histories $h$ such that $m \in h$ (the histories "passing through $m$")—we omit reference to the given tree, in this case.

*BT+AC* structures are introduced by the following definition as branching-time structures augmented by choices of agents and groups. In order to define them, we need to fix a countable set of atomic propositions $Atm$ and a finite set of agents $Agt = \{1, \ldots, n\}$. Before the definition some preliminary notation: given a binary relation $\mathcal{R}$ on a set of elements $X$ and an element $x$ of $X$, we define $\mathcal{R}(x) = \{y \in X : x\mathcal{R}y\}$.

*Definition 2.3.* A *BT+AC* structure is a tuple $B = (T, (\sim_{\langle m,J\rangle})_{m \in Mom, J \in 2^{Agt}}, v)$ where:

- $T$ is a tree;
- every $\sim_{\langle m,J\rangle}$ is an equivalence relation on its corresponding set of histories $H_m$ passing through $m$;
- $v : Atm \longrightarrow 2^{Mom}$ is a valuation function associating atoms with sets of moments;

and such that:

(B1) for all $m \in Mom$ and for all $h_1, \ldots, h_n \in H_m$: $\bigcap_{1 \le i \le n} \sim_{\langle m,\{i\}\rangle} (h_i) \neq \emptyset$;

(B2) for all $m \in Mom$ and for all $J \in 2^{Agt}$: $\sim_{\langle m,J\rangle} = \bigcap_{i \in J} \sim_{\langle m,\{i\}\rangle}$;

(B3) for all $m, m' \in Mom$ and for all $h, h' \in H_T$: if $m \prec m'$ and $h, h' \in H_{m'}$, then $h, h' \in H_m$ and $h \sim_{\langle m,Agt\rangle} h'$.

$h \sim_{\langle m,J\rangle} h'$ means that history $h$ is choice-equivalent to history $h'$ for group $J$ at moment $m$. Constraint B1 expresses the so-called assumption of *independence of choices*: if for every agent $i \in Agt$, $\sim_{\langle m,\{i\}\rangle}(h_i)$ is a possible choice for agent $i$ at moment $m$, then the intersection of all these choices is non-empty. More intuitively, this means that agents can never be deprived of choices due to the choices made by other agents. Constraint B2 just says that the collective choice of the group $J$ is equal to the intersection of the choices of all its individuals. Constraint B3 corresponds to the property of *no choice between undivided histories*. It captures the idea that if two histories come together in some future moment then, in the present, each agent does not have a choice between these two histories. This implies that if an agent can choose between two histories at a later stage, then she does not have a choice between them in the present.

We here define a subclass of *BT+AC*s under the assumption of the discreteness of time. This step is needed in order to relate them with CGSs in Section 5.

*Definition 2.4.* A structure $B = (T, (\sim_{\langle m,J\rangle})_{m \in Mom, J \in 2^{Agt}}, v)$ is discrete iff:

(B4) every history $h$ in $H_T$ is isomorphic to the set of natural numbers.

Given the discreteness of time assumption, for every moment in a history we can identify the successor moment along this history.

*Definition 2.5.* Let $B = (T, (\sim_{\langle m,J\rangle})_{m \in Mom, J \in 2^{Agt}}, v)$ be a discrete *BT+AC* structure and let $h \in H_T$. Then, $succ_h : h \longrightarrow h$ is the successor moment function for the history $h$ such that, for all $m, m' \in h$, $succ_h(m) = m'$ iff $m \prec m'$ and there is no $m'' \in h$ such that $m \prec m'' \prec m'$.

Constraint B4 in Definition 2.4 guarantees that the function $succ_h$ is well-defined. The seriality of the relation $\prec$ guarantees that $succ_h$ is total.

[3] In [7], they moreover provide a polynomial embedding of ATL into the 'strategic' variant of STIT by [16].

[4] Left linear means that for all $m, m', m'' \in Mom$, if $m' \prec m$ and $m'' \prec m$ then $m' = m''$ or $m' \prec m''$ or $m'' \prec m'$.

[5] Rooted means that there exists $m \in Mom$ such that for all $m' \in Mom$, $m \prec m'$ or $m = m'$.

## 3 DT-STIT$_n^{\mathrm{G}}$ LANGUAGE

We now introduce the language of discrete-time temporal group STIT logic DT-STIT$_n^{\mathrm{G}}$. This language, denoted by $\mathcal{L}_{\text{DT-STIT}_n^{\mathrm{G}}}(Atm, n)$, extends the language of CTL* by 'seeing-to-it-that' operators for all groups. It is defined by the following BNF:

$$\varphi, \psi \quad ::= \quad p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathrm{X}\varphi \mid \varphi \,\mathrm{U}\, \psi \mid \Box\varphi \mid [J]\varphi$$

where $p$ ranges over $Atm$ and $J$ ranges over $2^{Agt}$. When there is no risk of confusion, we simply write $\mathcal{L}_n^{\mathrm{G}}$ instead of $\mathcal{L}_{\text{DT-STIT}_n^{\mathrm{G}}}(Atm, n)$. The length $|\varphi|$ of the formula $\varphi$ is the number of occurrences of symbols in $\varphi$.

$\mathrm{X}\varphi$, $\varphi \,\mathrm{U}\, \psi$ and $\Box\varphi$ have a similar reading as in CTL*: $\mathrm{X}\varphi$ has to be read "$\varphi$ will be true in the next moment along the current history", $\varphi \,\mathrm{U}\, \psi$ has to be read "$\psi$ is true now or will be true at some moment in the future along the current history, and $\varphi$ has to hold until $\psi$", and $\Box\varphi$ has to be read "$\varphi$ is true in all possible histories starting in the current moment". $[J]\varphi$ has to be read "group $J$ sees to it that $\varphi$, regardless of what the agents outside $J$ choose".

Formulas of the language $\mathcal{L}_n^{\mathrm{G}}$ are evaluated with respect to a discrete *BT+AC* structure $B = (T, (\sim_{\langle m,J\rangle})_{m\in M, J\in 2^{Agt}}, v)$ and a moment-history pair $\langle m, h\rangle$ such that $m \in Mom$ and $h \in H_m$:

$B, \langle m,h\rangle \models p \iff m \in v(p)$

$B, \langle m,h\rangle \models \neg\varphi \iff B, \langle m,h\rangle \not\models \varphi$

$B, \langle m,h\rangle \models \varphi \wedge \psi \iff B, \langle m,h\rangle \models \varphi$ AND $B, \langle m,h\rangle \models \psi$

$B, \langle m,h\rangle \models \Box\varphi \iff \forall h' \in H_m : B, \langle m,h'\rangle \models \varphi$

$B, \langle m,h\rangle \models [J]\varphi \iff \forall h' \in H_m :$ IF $h \sim_{\langle m,J\rangle} h'$ THEN $B, \langle m,h'\rangle \models \varphi$

$B, \langle m,h\rangle \models \mathrm{X}\varphi \iff B, \langle succ_h(m), h\rangle \models \varphi$

$B, \langle m,h\rangle \models \varphi \,\mathrm{U}\, \psi \iff \exists m' \in h : m \leq m'$ AND $B, \langle m',h\rangle \models \psi$ AND $\forall m'' \in h :$ IF $m \leq m'' \prec m'$ THEN $B, \langle m'',h\rangle \models \varphi$

A formula $\varphi$ of the language $\mathcal{L}_n^{\mathrm{G}}$ is satisfiable relative to the class of discrete *BT+AC*s iff there exists a discrete *BT+AC* $B$ and a moment-history pair $\langle m,h\rangle$ such that $B, \langle m,h\rangle \models \varphi$. The formula $\varphi$ is valid relative to this class iff $\neg\varphi$ is not satisfiable.

## 4 CGS-BASED SEMANTICS FOR DT-STIT$_n^{\mathrm{G}}$

In this section, we provide a semantics for DT-STIT$_n^{\mathrm{G}}$ based on concurrent game structures (CGSs). We first remind the definition of this class of structures. Our presentation of CGSs slightly differs from the way CGSs are usually presented in the semantics for ATL and SL (see, e.g., [13, 21]). Specifically, we associate every joint action to a binary relation over states satisfying certain properties, while they use a transition function mapping every state and every joint action executable at this state to a successor state.

*Definition 4.1.* A concurrent game structure (CGS) is a tuple $M = (W, Act, (\mathcal{R}_\delta)_{\delta\in\mathcal{J}Act}, \mathcal{V})$ where:

- $W$ is a nonempty set of possible worlds or states;
- $Act$ is a set of names for atomic actions, with $\mathcal{J}Act = Act^n$ the corresponding set of names for joint actions and with elements of $\mathcal{J}Act$ denoted by $\delta, \delta', \ldots$;
- every $\mathcal{R}_\delta$ is a binary relation on $W$;
- $\mathcal{V} : W \longrightarrow 2^{Atm}$ is a valuation function;

and such that for every $w, v, u \in W$, $\delta \in \mathcal{J}Act$:

(C1) $\mathcal{R}_\delta$ is deterministic, i.e., for all $w \in W$ there is at most one $v$ such that $w\mathcal{R}_\delta v$;
(C2) if $\delta(1) \in C_1(w), \ldots, \delta(n) \in C_n(w)$ then $\mathcal{R}_\delta(w) \neq \emptyset$;
(C3) $\bigcup_{\delta\in\mathcal{J}Act} \mathcal{R}_\delta(w) \neq \emptyset$;

where $\delta(i)$ is the $i^{\text{th}}$ component of $\delta$ and $C_i(w) = \{a \in Act : \exists\delta \in \mathcal{J}Act$ s.t. $\mathcal{R}_\delta(w) \neq \emptyset$ and $\delta(i) = a\}$.

Constraint C1 expresses *joint action determinism*, namely, the fact that the outcome of the collective choice of all agents is uniquely determined. Constraint C2 corresponds to the *independence of choices* assumption in a way similar to Constraint B1 in Definition 2.3. According to Constraint C3, every state in a CGS has *at least one successor*, where the successor of a given state is a state which is reachable from the former via a collective choice of all agents. Notice that the set $C_i(w)$ in the previous definition corresponds to agent $i$'s *set of available actions* at state $w$, i.e., the set of actions that agent $i$ can choose at state $w$.

The previous notion of CGS is the one traditionally used in area of logics for multi-agent systems. In this paper, we consider a more general class of CGSs, called non-deterministic CGS, that better relate with *BT+AC* structures, as defined in Definition 2.3.

*Definition 4.2.* A *non-deterministic CGS* is like a CGS except that it does not necessarily satisfy Constraint C1 of joint action determinism in Definition 4.1.

The following definition introduces the concept of trace, as an infinite sequence of alternating states and joint actions such that a joint action is responsible for the transition from its preceding state to its subsequent state. In other words, a trace $\tau$ can be seen as an infinite sequence $w_1\delta_1 w_2\delta_2 w_3\delta_3 \ldots$ such that $w_k \in W$, $\delta_k \in \mathcal{J}Act$ and $w_k\mathcal{R}_{\delta_k} w_{k+1}$, for all $k > 0$.

*Definition 4.3.* Let $M = (W, Act, (\mathcal{R}_\delta)_{\delta\in\mathcal{J}Act}, \mathcal{V})$ be a CGS. A *trace* in $M$ is a pair $\tau = (\tau_\mathcal{S}, \tau_C)$ with $\tau_\mathcal{S} : \mathbb{N}^* \longrightarrow W$ and $\tau_C : \mathbb{N}^* \longrightarrow \mathcal{J}Act$ such that $\tau_\mathcal{S}(k)\mathcal{R}_{\tau_C(k)}\tau_\mathcal{S}(k+1)$ for all $k\mathbb{N}^*$. The set of all traces in $M$ is denoted by $Trace_M$.

Given a trace $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ and $k \in \mathbb{N}^*$, $\tau^{\geq k}$ is the trace $(\tau'_\mathcal{S}, \tau'_C)$ in $Trace_M$ such that, for all $\ell > 0$:

$$\tau'_\mathcal{S}(\ell) = \tau_\mathcal{S}(k + \ell - 1) \text{ and } \tau'_C(\ell) = \tau_C(k + \ell - 1).$$

In other words, if $\tau$ is the infinite sequence $w_1\delta_1 w_2\delta_2 w_3\delta_3 \ldots$, then $\tau^{\geq k}$ is the infinite sequence $w_k\delta_k w_{k+1}\delta_{k+1} \ldots$. More succinctly, $\tau^{\geq k}$ is the suffix of the sequence $\tau$ starting at the $k^{\text{th}}$ state in $\tau$.

The following definition introduces the concept of *choice equivalence*. The idea is that two traces are choice equivalent for a given group $J$ iff (i) the two traces have the same initial state, and (ii) the agents in the group make the same choices at the beginning of the two traces.

*Definition 4.4.* Two traces $\tau = (\tau_\mathcal{S}, \tau_C)$ and $\tau' = (\tau'_\mathcal{S}, \tau'_C)$ are *state-equivalent*, denoted by $\tau \equiv \tau'$, if and only if $\tau_\mathcal{S}(1) = \tau'_\mathcal{S}(1)$. Two traces $\tau = (\tau_\mathcal{S}, \tau_C)$ and $\tau' = (\tau'_\mathcal{S}, \tau'_C)$ are *choice-equivalent* for group $J \in 2^{Agt}$, denoted by $\tau \equiv_J \tau'$, if and only if $\tau \equiv \tau'$, and $\tau_C(1)(i) = \tau'_C(1)(i)$ for all $i \in J$.

Truth of a DT-STIT$_n^{\mathsf{G}}$ formula is evaluated with respect to a non-deterministic CGS $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ and a trace $\tau = (\tau_\mathcal{S}, \tau_C)$ in $M$, as follows:

$$\begin{aligned}
&M, \tau \models p && \Longleftrightarrow p \in \mathcal{V}(\tau_\mathcal{S}(1))\\
&M, \tau \models \neg\varphi && \Longleftrightarrow M, \tau \not\models \varphi\\
&M, \tau \models \varphi \wedge \psi && \Longleftrightarrow M, \tau \models \varphi \text{ AND } M, \tau \models \psi\\
&M, \tau \models \mathrm{X}\varphi && \Longleftrightarrow M, \tau^{\geq 2} \models \varphi\\
&M, \tau \models \varphi \,\mathrm{U}\, \psi && \Longleftrightarrow \exists k \in \mathbb{N}^* : M, \tau^{\geq k} \models \psi \text{ AND}\\
&&& \qquad \forall h \in \mathbb{N} : \text{IF } 1 \leq h < k \text{ THEN } M, \tau^{\geq h} \models \varphi\\
&M, \tau \models \Box\varphi && \Longleftrightarrow \forall \tau' \in Trace_M : \text{IF } \tau \equiv \tau' \text{ THEN } M, \tau' \models \varphi\\
&M, \tau \models [J]\varphi && \Longleftrightarrow \forall \tau' \in Trace_M : \text{IF } \tau \equiv_J \tau' \text{ THEN } M, \tau' \models \varphi
\end{aligned}$$

Validity and satisfiability of DT-STIT$_n^{\mathsf{G}}$ relative to CGSs and non-deterministic CGSs are defined in the usual way.

## 5 TREE-MODEL PROPERTY AND SEMANTIC EQUIVALENCE

Let $\mathcal{R}^*$, $\mathcal{R}^-$ and $\mathcal{R}^+$ be, respectively, the reflexive, transitive closure, the inverse and the transitive closure of $\mathcal{R} = \bigcup_{\delta \in \mathcal{J}Act} \mathcal{R}_\delta$.

*Definition 5.1.* Let $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ be a non-deterministic CGS. We say that:

- $M$ has a unique root iff there is a unique $w_0 \in W$ (called the *root*), such that, for every $v \in W$, $w_0 \mathcal{R}^* v$;
- $M$ has unique predecessors iff for every $v \in W$, the cardinality of $\mathcal{R}^-(v)$ is at most one;
- $M$ has no cycles iff $\mathcal{R}^+$ is irreflexive.

*Definition 5.2.* A non-deterministic CGS is tree-like if and only if it has a unique root, unique predecessors and no cycles.

The following lemma states that satisfiability relative to the class of non-deterministic CGSs with unique predecessors and no cycles is equivalent to satisfiability relative to the class of tree-like non-deterministic CGSs.

Lemma 5.3. *Let $\varphi \in \mathcal{L}_n^G$. Then, $\varphi$ is satisfiable relative to non-deterministic CGSs with unique predecessors and no cycles iff $\varphi$ is satisfiable relative to tree-like non-deterministic CGSs.*

Proof sketch. The right-to-left direction is clear. We prove the left-to-right direction. Let $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ be a non-deterministic CGS with unique predecessors and no cycles and let $\tau_0 = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ such that $M, \tau_0 \models \varphi$. Let $w_0 = \tau_\mathcal{S}(1)$. Let $M' = (W', Act, (\mathcal{R}'_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V}')$ the submodel of $M$ generated by the state $w_0$, that is:

- $W' = \{v \in W : w_0 \mathcal{R}^* v\}$,
- $\mathcal{R}'_\delta = \mathcal{R}_\delta \cap (W' \times W')$ for all $\delta \in \mathcal{J}Act$,
- $\mathcal{V}'(v) = \mathcal{V}(v)$ for all $v \in W'$.

Clearly, $M'$ is a tree-like CGS and $\tau_0 \in Trace_{M'}$. Moreover, it is easy to prove, by structural induction on $\varphi$, that $M', \tau_0 \models \varphi$. □

The following lemma states that satisfiability relative to the class of non-deterministic CGSs is equivalent to satisfiability relative to the class of non-deterministic CGSs with unique predecessors and no cycles.

Lemma 5.4. *Let $\varphi \in \mathcal{L}_n^G$. Then, $\varphi$ is satisfiable relative to non-deterministic CGSs iff $\varphi$ is satisfiable relative to non-deterministic CGSs with unique predecessors and no cycles.*

Proof sketch. The right-to-left direction of the lemma is clear. We prove the left-to-right direction.

Let $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ be a non-deterministic CGS and let $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ such that $M, \tau \models \varphi$.

We first define the set of tracks in $M$, denoted by $Track_M$, a track being a non-empty finite sequence $w_0 \delta_1 w_1 \ldots \delta_k w_k$ such that (i) $w_0 \in W$, (ii) $\delta_1 w_1 \ldots \delta_k w_k$ is a possibly finite sequence in $(\mathcal{J}Act \times W)^*$, and (iii) for every $0 \leq h \leq k-1$, $w_h \mathcal{R}_{\delta_{h+1}} w_{h+1}$. Elements of $Track_M$ are denoted by $\sigma, \sigma', \ldots$ For every $\sigma \in Track_M$, we denote by $\sigma[last]$ the last element in the sequence $\sigma$.

Given a trace $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ and $k > 0$, let $\tau^{\leq k}$ be the track $\tau_\mathcal{S}(1)\tau_C(1) \ldots \tau_C(k-1)\tau_\mathcal{S}(k)$.

We are going to transform $M$ into a new non-deterministic CGS $M' = (W', Act, (\mathcal{R}'_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V}')$ where:

- $W' = Track_M$;
- for all $\sigma, \sigma' \in W'$ and for all $\delta \in \mathcal{J}Act$, $\sigma \mathcal{R}'_\delta \sigma'$ iff $\sigma' = \sigma\delta v$ for some $v \in W$.
- for all $p \in Atm$ and for all $\sigma \in W'$, $p \in \mathcal{V}'(\sigma)$ iff $p \in \mathcal{V}(\sigma[last])$.

In other words, the model $M'$ is defined as follows: (i) its set of states coincides with the set of tracks in $M$, (ii) a joint action $\delta$ is responsible for the transition from the track $\sigma$ to the track $\sigma'$ iff $\sigma'$ is a possible continuation of the track $\sigma$ via the joint action $\delta$, and (ii) an atomic proposition $p$ is true at track $\sigma$ iff $p$ is true in the last state of $\sigma$. It is straightforward to verify that $M'$ is a CGS with unique predecessors and no cycles.

Let us define the function $f$ mapping traces in $M$ into traces in $M'$. Let $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ and $\tau' = (\tau'_\mathcal{S}, \tau'_C) \in Trace_{M'}$. Then, $f(\tau) = \tau'$ iff, for all $k > 0$: (i) $\tau'_\mathcal{S}(k) = \tau^{\leq k}$, and (ii) $\tau'_C(k) = \tau_C(k)$. It is routine to verify that $f$ so defined is a bijection.

By induction on the structure of $\varphi$, it can be shown that "$M, \tau \models \varphi$ iff $M', f(\tau) \models \varphi$". Hence, $M', f(\tau) \models \varphi$. □

The following theorem follows straightforwardly from Lemma 5.3 and Lemma 5.4. It highlights that DT-STIT$_n^{\mathsf{G}}$ interpreted over the CGS semantics satisfies the tree-model property.

Theorem 5.5. *Let $\varphi \in \mathcal{L}_n^G$. Then, $\varphi$ is satisfiable relative to non-deterministic CGSs iff $\varphi$ is satisfiable relative to tree-like non-deterministic CGSs.*

The final result of this section is a lemma stating that satisfiability for DT-STIT$_n^{\mathsf{G}}$ relative to the class of *BT+AC* structures is equivalent to satisfiability for DT-STIT$_n^{\mathsf{G}}$ relative to the class of tree-like non-deterministic CGSs.

Lemma 5.6. *Let $\varphi \in \mathcal{L}_n^G$. Then, $\varphi$ is satisfiable relative to BT+AC structures iff $\varphi$ is satisfiable relative to tree-like non-deterministic CGSs.*

Proof sketch. We first prove the left-to-right direction. Let $B = (T, (\sim_{\langle m,J \rangle})_{m \in M, J \in 2^{Agt}}, v)$ be a *BT+AC* structure where $T = (Mom, <)$ is its corresponding tree. Moreover, let $m_0 \in Mom$ be a moment and $h_0 \in H_{m_0}$ a history passing through $m_0$ such that $B, \langle m_0, h_0 \rangle \models \varphi$.

We are going to transform $B$ into a new structure $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ whose components are defined as follows:

- $W = Mom$;
- $Act = \bigcup_{m \in Mom, i \in Agt} H_m / \sim_{\langle m, \{i\} \rangle}$;
- for all $m, m' \in W$ and for all $\delta \in \mathcal{J}Act$, $m\mathcal{R}_\delta m'$ iff there exists $h \in H_m$ such that $succ_h(m) = m'$ and $\delta(i) =\sim_{\langle m, \{i\} \rangle}(h)$ for all $i \in Agt$;
- for all $p \in Atm$ and for all $m \in W$, $p \in \mathcal{V}(m)$ iff $p \in v(m)$.

It is routine to verify that $M$ is a tree-like non-deterministic CGS. Furthermore, by induction on the structure of $\varphi$, it is easy to check that $M, \tau \models \varphi$, where $\tau = (\tau_\mathcal{S}, \tau_C)$ is the trace in $Trace_M$ such that, for all $k > 0$: (i) $\tau_\mathcal{S}(k) = g(k)$, and (ii) $\tau_C(k) = f_{g(k)}(\sim_{\langle g(k), Agt \rangle}(h_0))$ and where the function $g : \mathbb{N}^* \longrightarrow Mom$ is defined inductively as follows: (iii) $g(1) = m_0$, and (iv) for all $k > 0$, $g(k+1) = succ_{h_0}(g(k))$.

Let us now prove the right-to-left direction. Let $M = (W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V})$ be a tree-like non-deterministic CGS and let $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$ such that $M, \tau \models \varphi$. We are going to transform $M$ into a new structure $B = (T, (\sim_{\langle m, J \rangle})_{m \in M, J \in 2^{Agt}}, v)$ with $T = (Mom, \prec)$. First, we define $Mom$ and $\prec$ in the pair $T$:

- $Mom = W$;
- for all $w, v \in Mom$, $w \prec v$ iff $w\mathcal{R}^+ v$.

Since $M$ is tree-like, it is easy to verify that $T$ so defined is a tree. Moreover, because of the tree-likeness of $M$, we can define a function $t : Trace_M \longrightarrow H_T$ such that for all $\tau = (\tau_\mathcal{S}, \tau_C) \in Trace_M$:

$$t(\tau) = \{w \in W : w\mathcal{R}^+ \tau_\mathcal{S}(1)\} \cup \{\tau_\mathcal{S}(k) : k \in \mathbb{N}^*\}.$$

Then, we define the remaining components of the tuple $B$:

- for all $w \in Mom$, for all $J \in 2^{Agt}$ and for all $h, h' \in H_w$, $h \sim_{\langle w, J \rangle} h'$ iff there exists $\delta, \delta' \in \mathcal{J}Act$ and $v, u \in W$ such that $\delta_J = \delta'_J$, $v \in h$, $u \in h'$, $w\mathcal{R}_\delta v$ and $w\mathcal{R}_{\delta'} u$;
- for all $p \in Atm$ and for all $w \in Mom$, $p \in v(w)$ iff $p \in \mathcal{V}(w)$;

where $\delta_J = \delta'_J$ iff $\delta(i) = \delta'(i)$ for all $i \in J$.

It is easy to verify that $B$ so defined is a *BT+AC* structure.

Furthermore, by induction on the structure of $\varphi$, it is easy to check that $B, \langle m, h \rangle \models \varphi$, for $m = \tau_\mathcal{S}(1)$ and $h = t(\tau)$. □

The following theorem is a direct consequence of Theorem 5.5 and Lemma 5.6.

Theorem 5.7. *Let $\varphi \in \mathcal{L}_n^G$. Then, $\varphi$ is satisfiable relative to BT+AC structures iff $\varphi$ is satisfiable relative to non-deterministic CGSs.*

## 6 INDIVIDUAL FRAGMENT

The satisfiability problem of $\text{DT-STIT}_n^G$ is undecidable if $n > 2$. To show this, consider the satisfiability-preserving translation from group STIT's language to $\mathcal{L}_n^G$ that replaces all propositional variables $p$ with X$p$. Since group STIT has been proved in [15] to be undecidable, $\text{DT-STIT}_n^G$ is also undecidable. This undecidability result holds even if the language is restricted to groups of cardinality at most two. Therefore, to obtain a decidable fragment, we restrict the language of $\text{DT-STIT}_n^G$ to groups that are singletons. We call this fragment the discrete-time temporal individual STIT logic, $\text{DT-STIT}_n$ for short. For the sake of simplicity, we write $[i]\,\varphi$ instead of $[\{i\}]\,\varphi$. Moreover, assuming that $n \geq 2$, we omit the historical necessity, since [3] proved that it can be defined by $\Box\varphi \doteq [i]\,[j]\,\varphi$ for some arbitrary distinct $i, j \in Agt$. The resulting language $\mathcal{L}_{\text{DT-STIT}_n}(Atm, n)$ is defined by the following BNF:

$$\varphi \quad ::= \quad p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathrm{X}\varphi \mid \varphi \,\mathrm{U}\, \psi \mid [i]\varphi$$

where $p$ range over $Atm$ and $i$ over $Agt$. When there is no risk of confusion, we simply write $\mathcal{L}_n$ instead of $\mathcal{L}_{\text{DT-STIT}_n}(Atm, n)$. We define the usual closure property on sets of formulas. Formally, a set $\Sigma \subseteq \mathcal{L}_n$ is *closed* iff:

1. $\forall \varphi \in \Sigma$, if $\psi$ is a subformula of $\varphi$ then $\psi \in \Sigma$,
2. $\forall \varphi \in \Sigma$, if $\varphi = \neg\psi$ then $\psi \in \Sigma$ else $\neg\varphi \in \Sigma$, and
3. $\forall \varphi \,\mathrm{U}\, \psi \in \Sigma$, $\mathrm{X}(\varphi \,\mathrm{U}\, \psi) \in \Sigma$.

For all formula $\varphi_0 \in \mathcal{L}_n$, $\mathrm{Cl}(\varphi_0)$ denotes the least closed set containing $\varphi_0$. It can easily be proved that $|\mathrm{Cl}(\varphi_0)|$ is linear in $|\varphi_0|$.

## 7 PSEUDO-MODEL SEMANTICS

In this section, we describe an alternative semantics for $\text{DT-STIT}_n$ that makes it easy for an automaton to check the satisfiability of a formula. This new semantics can be broken down into two components: the local pseudo-models which correspond to worlds in the CGS semantics and the pseudo-models which are infinite trees whose branches correspond to traces in the CGS semantics. Through this section, $\Sigma$ denotes an arbitrary closed set of formulas. In Section 8.1, the construction of a pseudo-model from a non-deterministic CGS provides some intuitions for the abstract definitions of the pseudo-model semantics.

### 7.1 Local pseudo-models

*Definition 7.1.* A subset $h \subseteq \Sigma$ is *maximal locally consistent* iff:

1. $\forall \neg\varphi \in \Sigma$, $\varphi \in h$ iff $\neg\varphi \in h$,
2. $\forall \varphi \wedge \psi \in \Sigma$, $\varphi \wedge \psi \in h$ iff $\varphi \in h$ and $\psi \in h$,
3. $\forall \varphi \,\mathrm{U}\, \psi \in \Sigma$, $\varphi \,\mathrm{U}\, \psi \in h$ iff $\psi \in h$ or $\{\varphi, \mathrm{X}(\varphi \,\mathrm{U}\, \psi)\} \subseteq h$, and
4. $\forall\, [i]\,\varphi \in h$, $\varphi \in h$.

The set of maximal locally consistent subsets of $\Sigma$ is denoted by $\mathcal{H}_\Sigma$. When $\Sigma = \mathrm{Cl}(\varphi_0)$ we simply write $\mathcal{H}_{\varphi_0}$.

For all $h_1, h_2 \in \mathcal{H}_\Sigma$ and all $i \in Agt$, we say that $h_1$ and $h_2$ are $[i]$-compatible iff for all $\varphi \in \Sigma$ such that $\varphi \in Atm$ or $\varphi = [i]\,\psi$ for some $\psi \in \mathcal{L}_n$, $\varphi \in h_1$ iff $\varphi \in h_2$. We say that $h_1$ and $h_2$ are $\Box$-compatible iff they are $[i]$-compatible for all $i \in Agt$.

*Definition 7.2.* A *local pseudo-model* is a tuple $(H, \sim, h_0)$ where $H$ is a subset of $\mathcal{H}_\Sigma$, $\sim$ is a function assigning to each agent $i$ an equivalence relation $\overset{i}{\sim}$ over $H$ and $h_0$ is a designated element of $H$. It must satisfy the following conditions:

1. for all $i \in Agt$ and all $h_1, h_2 \in H$, if $h_1 \overset{i}{\sim} h_2$ then $h_1$ and $h_2$ are $[i]$-compatible,
2. for all $[i]\,\varphi \in \Sigma$ and all $h_1 \in H$, if $[i]\,\varphi \notin h_1$ then there is $h_2 \in \overset{i}{\sim}(h_1)$ such that $\varphi \notin h_2$, and
3. for all $h_1, \ldots, h_n \in H$, $\cap_{i \in Agt} \overset{i}{\sim}(h_i) \neq \emptyset$,

where $\overset{i}{\sim}(h_1) \doteq \left\{h_2 \in H \,\middle|\, h_1 \overset{i}{\sim} h_2\right\}$. The set of local pseudo-models for $\Sigma$ is denoted by $\mathbb{L}_\Sigma$. When $\Sigma = \mathrm{Cl}(\varphi_0)$ we simply write $\mathbb{L}_{\varphi_0}$.

Lemma 7.3. *The cardinality of $\mathbb{L}_\Sigma$ is double exponential in $|\Sigma|$.*

Proof sketch. Let $B_k$ denote the $k^{\text{th}}$ Bell number. It can easily be checked that $|\mathbb{L}_\Sigma|$ is bounded by $2^{2^{|\Sigma|}} B_{|\Sigma|}^n 2^{|\Sigma|}$. □

## 7.2 Pseudo-models

For any alphabet $\mathbb{A}$, we write $\mathbb{A}^*$ and $\mathbb{A}^\omega$ to denote respectively the set of all finite sequences over $\mathbb{A}$ and the set all infinite sequences over $\mathbb{A}$. We write $\epsilon$ to denote an empty sequence. For all $\sigma \in \mathbb{A}^*$, we write $|\sigma|$ to denote the *length* of $\sigma$. For all finite or infinite sequences $\sigma$ and all $k > 0$, we write $\sigma^k$, $\sigma^{\leq k}$ and $\sigma^{\geq k}$ to denote respectively the $k^{\text{th}}$ element in $\sigma$, the prefix of $\sigma$ of length $k$ and the suffix of $\sigma$ starting at the $k^{\text{th}}$ element. By convention, $\sigma^{\leq 0} = \epsilon$.

A *pre-model* on $\Sigma$ is an infinite tree[6] $t : (\mathcal{H}_\Sigma)^* \longrightarrow \mathbb{L}_\Sigma \cup \{\text{Nop}\}$ over the alphabet $\mathcal{H}_\Sigma$, labeled with $\mathbb{L}_\Sigma \cup \{\text{Nop}\}$, where Nop is any fixed mathematical object such that $\text{Nop} \notin \mathbb{L}_\Sigma$. An infinite sequence $\sigma \in (\mathcal{H}_\Sigma)^\omega$ is a *path* in $t$. We say that:

- $\sigma$ is an *active path of* $t$ iff for all prefixes $\theta$ of $\sigma$, $t(\theta) \neq \text{Nop}$;
- $\sigma$ is a $\diamond$*-path of* $t$ iff there is a finite sequence $\theta \in (\mathcal{H}_\Sigma)^*$, called the initial point of $\sigma$, such that:
  - $\theta\sigma$ is an active path of $t$,
  - $\sigma^1 \in H_0$ for $(H_0, \sim_0, h_{00}) = t(\theta)$, and
  - for all $k \geq 1$, $\sigma^{k+1} = h_{0k}$ for $(H_k, \sim_k, h_{0k}) = t\left(\theta\sigma^{\leq k}\right)$;
- $\sigma$ is a $\square$*-compatible support* of an active path $\theta$ of $t$ iff for all $k > 0$, $\sigma^k$ and $\theta^k$ are $\square$-compatible.

For all $S \subseteq \mathcal{L}_n$, let $S^{\text{X}}$ denote the set of formulas $\varphi$ such that $\text{X}\varphi \in S$ or $\neg\text{X}\neg\varphi \in S$.

*Definition 7.4.* An infinite sequence $\sigma \in (\mathcal{H}_\Sigma)^\omega$ is *fulfilling* iff for all $k > 0$:

(1) $\left(\sigma^k\right)^{\text{X}} \subseteq \sigma^{k+1}$, and
(2) for all $\varphi \cup \psi \in \sigma^k$, there is $\ell \geq k$ such that $\psi \in \sigma^\ell$.

*Definition 7.5.* A pre-model $t$ on $\Sigma$ is a *pseudo-model* on $\Sigma$ iff:

(1) $t(\epsilon) \neq \text{Nop}$ and for all $\sigma \in (\mathcal{H}_\Sigma)^*$ and $h \in \mathcal{H}_\Sigma$, $t(\sigma h) \neq \text{Nop}$ iff there is $(H, \sim, h_0) \in \mathbb{L}_\Sigma$ such that $t(\sigma) = (H, \sim, h_0)$ and $h \in H$,
(2) all $\diamond$-path of $t$ are fulfilling, and
(3) for all active path $\sigma$ of $t$, there is a $\square$-compatible support of $\sigma$ that is fulfilling.

A pseudo-model $t$ on $\Sigma$ *satisfies* a formula $\varphi_0 \in \Sigma$ iff $t(\epsilon) = (H_\epsilon, \sim_\epsilon, h_{0\epsilon})$ and $\varphi_0 \in h_{0\epsilon}$ for some $(H_\epsilon, \sim_\epsilon, h_{0\epsilon}) \in \mathbb{L}_\Sigma$. A formula $\varphi_0 \in \mathcal{L}_n$ is *satisfiable* in the pseudo-model semantics iff there is a pseudo-model $t$ on $\text{Cl}(\varphi_0)$ that satisfies $\varphi_0$.

The following lemma states that if there is a fulfilling $\square$-compatible support of a path, then this support is unique. It can easily be proved by a standard induction on the formula $\varphi$.

Lemma 7.6. *Let $\sigma_1$ and $\sigma_2$ be infinite fulfilling sequences such that for all $k > 0$, $\sigma_1^k$ and $\sigma_2^k$ are $\square$-compatible. Then for all $\varphi \in \Sigma$ and all $k > 0$, $\varphi \in \sigma_1^k$ iff $\varphi \in \sigma_2^k$.*

# 8 EQUIVALENCE OF THE SEMANTICS

In this section, we prove the following theorem that states the equivalence between the pseudo-model semantics and the non-deterministic CGS semantics. Since the pseudo-model semantics is used in the decision procedure for DT-STIT$_n$ satisfiability, the proof of the left-to-right direction is called *completeness* and the proof of the right-to-left direction is called *soundness.*

[6] Given a set $S$, let $<$ be the strict lexicographic order on $S^*$. By Definition 2.1, $(S^*, <)$ is a tree. We abusively identify labeled trees with their labeling function.

Theorem 8.1. *A formula $\varphi_0 \in \mathcal{L}_n$ is satisfiable in the non-deterministic CGS semantics iff it is satisfiable in the pseudo-model semantics.*

## 8.1 Completeness

Let $M = \left(W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V}\right)$ be a non-deterministic CGS, $\tau_0$ a trace in $M$ and $\varphi_0 \in \mathcal{L}_n$ a formula such that $M, \tau_0 \vDash \varphi_0$. We will construct a pseudo-model on $\text{Cl}(\varphi_0)$ satisfying $\varphi_0$.

Firstly, we need to associate a local pseudo-model to each world in $W$. This step is analogous to the filtration in [3] except that we need to consider traces. Formally, we define the function $\Phi$ such that $\Phi(\tau) = \{\varphi \in \text{Cl}(\varphi_0) \mid M, \tau \vDash \varphi\}$ for all $\tau \in Trace_M$. Then we associate to each world $w \in W$ a pair $(H_w, \sim_w)$ where:

- $H_w \doteq \left\{h \in \mathcal{H}_{\varphi_0} \,\middle|\, \exists \tau \in Trace_M, \tau_{\mathcal{S}}(1) = w \text{ and } \Phi(\tau) = h\right\}$;
- $\sim_w$ is the function assigning to each $i \in Agt$ the equivalence relation $\overset{i}{\sim}_w$ over $H_w$ such that for all $h_1, h_2 \in H_w$, $h_1 \overset{i}{\sim}_w h_2$ iff there is $\tau_1, \tau_2 \in Trace_M$ such that $\tau_{1\mathcal{S}}(1) = \tau_{2\mathcal{S}}(1) = w$, $\tau_{1C}(1)(i) = \tau_{2C}(1)(i)$, $\Phi(\tau_1) = h_1$ and $\Phi(\tau_2) = h_2$.

Lemma 8.2. *For all $w \in W$ and all $h \in H_w$, $(H_w, \sim_w, h)$ is a local pseudo-model.*

Proof sketch. The proofs for conditions (1) and (2) of Definition 7.2 are straightforward. For (3), let $h_1, \ldots, h_n \in H_w$. There is $\tau_1, \ldots, \tau_n \in Trace_M$ such that for all $i \in Agt$, $\tau_{i\mathcal{S}}(1) = w$ and $\Phi(\tau_i) = h_i$. Construct $\delta \in \mathcal{J}Act$ such that for all $i \in Agt$, $\delta(i) = \tau_{iC}(1)(i)$. By C2, there is $x \in W$ such that $w\ \mathcal{R}_i\ x$. Hence by C3, there is $\tau_\cap \in Trace_M$ such that $\tau_{\cap\mathcal{S}}(1) = w$ and $\tau_{\cap C}(1) = \delta$. Clearly, for all $i \in Agt$, $h_i \overset{i}{\sim}_w \Phi(\tau_\cap)$. □

Secondly, we select traces in $M$ that will correspond to the active paths in the pseudo-model. Assuming that $\text{Nop} \notin Trace_M$, we construct inductively the functions $r : \left(\mathcal{H}_{\varphi_0}\right)^* \longrightarrow Trace_M \cup \{\text{Nop}\}$ and $s : \left(\mathcal{H}_{\varphi_0}\right)^* \times \mathcal{H}_{\varphi_0} \longrightarrow Trace_M \cup \{\text{Nop}\}$ as follows:

- $r(\epsilon) \doteq \tau_0$ and
- for all $\sigma \in \left(\mathcal{H}_{\varphi_0}\right)^*$ and all $h \in \mathcal{H}_{\varphi_0}$:
  - if $r(\sigma) = \text{Nop}$ then $s(\sigma, h) \doteq \text{Nop}$;
  otherwise, let $r(\sigma) = \tau_\sigma = (\tau_{\sigma\mathcal{S}}, \tau_{\sigma C})$ and
  - if $h \notin W_{\tau_{\sigma\mathcal{S}}(1)}$ then $s(\sigma, h) \doteq \text{Nop}$;
  - else if $\Phi(\tau_\sigma) = h$ then $s(\sigma, h) \doteq \tau_\sigma$;
  - otherwise, set $s(\sigma, h) \doteq \tau_{\sigma h}$ for some arbitrary $\tau_{\sigma h} = (\tau_{\sigma h\mathcal{S}}, \tau_{\sigma hC})$ such that $\tau_{\sigma h\mathcal{S}}(1) = \tau_{\sigma\mathcal{S}}(1)$ and $\Phi(\tau_{\sigma h}) = h$, which exists by construction;
- if $s(\sigma, h) = \text{Nop}$ then $r(\sigma h) \doteq \text{Nop}$ else $r(\sigma h) \doteq s(\sigma, h)^{\geq 2}$.

If $r(\sigma) = \tau = (\tau_{\mathcal{S}}, \tau_C)$, we write $r_{\mathcal{S}}(\sigma)$ and $r_C(\sigma)$ to denote respectively $\tau_{\mathcal{S}}$ and $\tau_C$. The notation is similar for $s$.

Finally, the pre-model $t$ on $\text{Cl}(\varphi_0)$ is constructed such that for all $\sigma \in \left(\mathcal{H}_{\varphi_0}\right)^*$, if $r(\sigma) = \text{Nop}$ then $t(\sigma) = \text{Nop}$ else $t(\sigma) = \left(H_{r_{\mathcal{S}}(\sigma)(1)}, \sim_{r_{\mathcal{S}}(\sigma)(1)}, \Phi(r(\sigma))\right)$.

Lemma 8.3. *$t$ is a pseudo-model on $Cl(\varphi_0)$.*

Proof sketch. We only prove condition (3) of Def. 7.5, the other ones being similar or straightforward. Let us say that a path $\sigma$ *matches* a trace $\tau$ iff for all $k > 0$, $\sigma^k = \Phi\left(\tau^{\geq k}\right)$. Obviously, in such a case, $\sigma$ is fulfilling. Let $\sigma$ be an active path of $t$. Define

$\tau = (\tau_{\mathcal{S}}, \tau_C)$ such that for all $k > 0$, $\tau_{\mathcal{S}}(k) = r_S\left(\sigma^{\leq k-1}\right)(1)$ and $\tau_C(k) = s_C\left(\sigma^{\leq k-1}, \sigma^k\right)(1)$. It can easily be checked that for all $k > 0$, $\Phi(\tau^{\geq k})$ and $\sigma^k$ are □-compatible. Hence there is a fulfilling □-compatible support of $\sigma$. □

Since $t$ satisfies $\varphi_0$, we have proved the left-to-right direction of Theorem 8.1.

## 8.2 Soundness

Let $\varphi_0$ be a formula in $\mathcal{L}_n$ and $t$ a pseudo-model on $\mathrm{Cl}(\varphi_0)$ satisfying $\varphi_0$. We will construct a non-deterministic CGS satisfying $\varphi_0$.

Define $W \doteq \left\{\sigma \in \left(\mathcal{H}_{\varphi_0}\right)^* \,\middle|\, t(\sigma) \neq \mathrm{Nop}\right\}$ and $Act \doteq 2^{\mathcal{H}_{\varphi_0}}$ and construct the tuple $M = \left(W, Act, (\mathcal{R}_\delta)_{\delta \in \mathcal{J}Act}, \mathcal{V}\right)$ such that for all $\sigma_1, \sigma_2 \in W$:

- for all $\delta \in \mathcal{J}Act$, $\sigma_1 \;\mathcal{R}_\delta\; \sigma_2$ iff there is $h \in \mathcal{H}_{\varphi_0}$ such that $\sigma_2 = \sigma_1 h$ and for all $i \in Agt$, $\delta(i) = \overset{i}{\sim}_1(h)$;
- for all $p \in Atm$, $p \in \mathcal{V}(\sigma_1)$ iff $p \in h_{01}$;

where $(H_1, \sim_1, h_{01}) = t(\sigma_1)$. It can easily be checked that $M$ is a non-deterministic CGS.

Then to each trace $\tau = (\tau_{\mathcal{S}}, \tau_C)$ of $M$, we associate the active path $\sigma_\tau$ and the integer $\ell_\tau$ such that for all $k > 0$, $\tau_{\mathcal{S}}(k) = \sigma^{\leq k+\ell_\tau}$. The following truth lemma can be proved by structural induction on $\varphi$.

Lemma 8.4. *For all formulas $\varphi \in Cl(\varphi_0)$, all traces $\tau = (\tau_{\mathcal{S}}, \tau_C)$ of $M$ and all fulfilling □-compatible supports $\theta$ of $\sigma_\tau$, $M, \tau \models \varphi$ iff $\varphi \in \theta^{\ell_\tau+1}$.*

Now, to prove the right-to-left direction of Theorem 8.1, it suffices to construct inductively the ◇-path $\sigma$ with initial point $\epsilon$ such that for all $k > 0$, $\sigma^k \doteq h_{0k}$ where $(H_k, \sim_k, h_{0k}) = t\left(\sigma^{\leq k-1}\right)$. By Lemma 7.6, $\sigma$ is its own □-compatible support. Moreover, there is a trace $\tau_0$ in $M$ such that $\sigma_{\tau_0} = \sigma$ and $\ell_{\tau_0} = 0$. Therefore, by Lemma 8.4, $M, \tau_0 \models \varphi_0$.

# 9 DECISION PROCEDURE

We propose a decision procedure for the satisfiability problem of DT-STIT$_n$. Given a formula $\varphi_0 \in \mathcal{L}_n$, this procedure constructs an automaton on infinite trees and returns whether there exists a tree that is recognized by this automaton. We prove that the procedure can be executed in double exponential time in $|\varphi_0|$ and that $\varphi_0$ is DT-STIT$_n$ satisfiable if and only if the procedure returns true.

## 9.1 Automata

Given a formula $\varphi_0 \in \mathcal{L}_n$, we construct an automaton that recognize exactly the pseudo-models on $\mathrm{Cl}(\varphi_0)$ satisfying $\varphi_0$. This automaton is the product of three automata: one for each condition of Definition 7.5. We first recall some basic notions about automata.

Given an alphabet $\mathbb{A}$, a non-deterministic Büchi word automaton over $\mathbb{A}$ is a tuple $\mathcal{A} = (\mathcal{S}, S_0, \rho, \mathcal{F})$ where $\mathcal{S}$ is the set of states of the automaton, $S_0 \in \mathcal{S}$ is the initial state, $\rho : \mathcal{S} \times \mathbb{A} \longrightarrow 2^{\mathcal{S}}$ is a non-deterministic transition function and $\mathcal{F} \subseteq \mathcal{S}$ is the termination condition. Given an infinite word $\sigma \in \mathbb{A}^\omega$, a *run* of $\mathcal{A}$ on $\sigma$ is a word $r \in \mathcal{S}^\omega$ such that $r^1 = S_0$ and for all $k \geq 1$, $r^{k+1} \in \rho(r^k, \sigma^k)$. The set of states occurring infinitely often in a run $r$ is denoted by $\inf(r)$. A word $\sigma$ is accepted by $\mathcal{A}$ iff there is a run $r$ of $\mathcal{A}$ on $\sigma$ such that $\inf(r) \cap \mathcal{F} \neq \emptyset$.

A deterministic Streett tree automaton over $\mathbb{A}$ is a tuple $\mathcal{A} = (\mathcal{S}, S_0, \rho, \mathcal{F})$ similar to a non-deterministic Büchi word automaton except that $\rho : \mathcal{S} \times \mathbb{A} \longrightarrow \mathcal{S}^n$ is a partial function that assigns an $n$-ary tuples of states and $\mathcal{F} \subseteq 2^{\mathcal{S}} \times 2^{\mathcal{S}}$ is a set of pairs of sets of states. Given a ordered set $\mathbb{I}$ of cardinality $n$ and an infinite tree $t : \mathbb{I}^* \longrightarrow \mathbb{A}$, a run of $\mathcal{A}$ on $t$ is a tree $t_r : \mathbb{I}^* \longrightarrow \mathcal{S}$ such that $t_r(\epsilon) = S_0$ and for all $\sigma \in \mathbb{I}^*$, $(t_r(\sigma\alpha))_{\alpha \in \mathbb{I}} = \rho(t_r(\sigma), t(\sigma))$. For all branches $\sigma \in \mathcal{S}^\omega$ of $t_r$, the set of states occurring infinitely often in $\sigma$ is denoted by $\inf(\sigma)$. A tree $t$ is accepted by $\mathcal{A}$ iff there is a run $t_r$ of $\mathcal{A}$ on $t$ such that for any branch $\sigma$ of $t_r$ and any pair $(A, B) \in \mathcal{F}$, if $\inf(\sigma) \cap A \neq \emptyset$ then $\inf(\sigma) \cap B \neq \emptyset$.

*9.1.1 Automaton for condition* (1) *of Def. 7.5.* The deterministic Streett tree automaton $\mathcal{A}_1 = (\mathcal{S}_1, S_{01}, \rho_1, \mathcal{F}_1)$ is defined such that $\mathcal{S}_1 \doteq \{\mathrm{def}, \mathrm{Nop}\}$, $S_{01} \doteq \mathrm{def}$, $\mathcal{F}_1 \doteq \emptyset$ and $\rho_1(S_\sigma, t(\sigma)) = (S_{\sigma h})_{h \in \mathcal{H}_{\varphi_0}}$ iff one of the following conditions holds:

- $S_\sigma = \mathrm{def}$, $t(\sigma) = (H, \sim, h_0)$ for some $(H, \sim, h_0) \in \mathbb{L}_{\varphi_0}$ and for all $h \in \mathcal{H}_{\varphi_0}$, $S_{\sigma h} = \begin{cases} \mathrm{def} & \text{if } h \in H \\ \mathrm{Nop} & \text{otherwise} \end{cases}$;
- $S_\sigma = \mathrm{Nop}$, $t(\sigma) = \mathrm{Nop}$ and for all $h \in \mathcal{H}_{\varphi_0}$, $S_{\sigma h} = \mathrm{Nop}$.

The following lemma is straightforward.

Lemma 9.1. *A pre-model $t$ on $Cl(\varphi_0)$ satisfies condition* (1) *of Def. 7.5 iff it is accepted by $\mathcal{A}_1$.*

*9.1.2 Automaton for condition* (2) *of Def. 7.5.* Define the deterministic Streett tree automaton $\mathcal{A}_2 = (\mathcal{S}_2, S_{02}, \rho_2, \mathcal{F}_2)$ where $\mathcal{S}_2 \doteq 2^{\mathrm{Cl}(\varphi_0)} \times 2^{\mathrm{Cl}(\varphi_0)}$, $S_{02} \doteq (\{\varphi_0\}, \emptyset)$, $\mathcal{F}_2 \doteq \{(\mathcal{S}_2, \{(c, e) \in \mathcal{S}_2 \mid e = \emptyset\})\}$ and $\rho_2((c_\sigma, e_\sigma), t(\sigma)) = ((c_{\sigma h}, e_{\sigma h}))_{h \in \mathcal{H}_{\varphi_0}}$ iff one of the following conditions holds:

- $t(\sigma) = (H, \sim, h_0)$ for some $(H, \sim, h_0) \in \mathbb{L}_{\varphi_0}$, $c_\sigma \subseteq h_0$ and for all $h \in \mathcal{H}_{\varphi_0}$ $c_{\sigma h} = h^{\mathrm{X}}$ and:

$$e_{\sigma h} = \begin{cases} \{\psi \mid \psi \notin h_0 \text{ and } \exists \varphi, \varphi \,\mathrm{U}\, \psi \in h_0\} & \text{if } h = h_0 \text{ and } e_\sigma = \emptyset \\ e_\sigma \setminus h_0 & \text{if } h = h_0 \text{ and } e_\sigma \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$$

- $t(\sigma) = \mathrm{Nop}$ and for all $h \in \mathcal{H}_{\varphi_0}$, $c_{\sigma h} = \emptyset$ and $e_{\sigma h} = e_\sigma$.
- $t(\sigma) = (H, \sim, h_0)$ for some $(H, \sim, h_0) \in \mathbb{L}_{\varphi_0}$, $c_\sigma \setminus h_0 \neq \emptyset$ and for all $h \in \mathcal{H}_{\varphi_0}$, $c_{\sigma h} = e_{\sigma h} = \mathrm{Cl}(\varphi_0)$.

Intuitively, for each state $(c, e)$, $c$ is the set of formulas that must be satisfied at the current state, whereas $e$ is the set of formulas that must be eventually satisfied.

Lemma 9.2. *If a pseudo-model $t$ on $Cl(\varphi_0)$ satisfies $\varphi_0$ then $t$ is accepted by $\mathcal{A}_2$.*

Proof sketch. Let $t$ be a pseudo-model satisfying $\varphi_0$. Since $\rho_2$ is total there is a run $t_r$ of $\mathcal{A}_2$ on $t$. Let $\sigma \in \left(\mathcal{H}_{\varphi_0}\right)^\omega$ be an arbitrary path in $t$. We will prove that the branch in $t_r$ corresponding to $\sigma$ satisfies the Streett condition $\mathcal{F}_2$. For all $k > 0$, let $(c_k, e_k) \doteq t_r(\sigma^{\leq k})$. Moreover, if $t(\sigma^{\leq k}) \neq \mathrm{Nop}$ let $(H_k, \sim_k, h_{0k}) \doteq t(\sigma^{\leq k})$. The following properties can be proved by induction on $k$:

$$\forall k > 0, \text{ if } t(\sigma^{\leq k}) \neq \mathrm{Nop} \text{ then } c_k \subseteq h_{0k} \quad (1)$$

$$\forall k > 0, \text{ if } e_k \neq \emptyset \text{ then } t(\sigma^{\leq k}) \neq \mathrm{Nop} \quad (2)$$

Now suppose that there is only a finite number of integer $k$ such that $e_k = \emptyset$. Since $e_0 = \emptyset$, there is a greatest integer $\ell$ such that $e_\ell = \emptyset$. Since $e_{\ell+1} \neq \emptyset$, $t\left(\sigma^{\leq \ell}\right) \neq$ Nop and by (1), for all $\psi \in e_{\ell+1}$, $\psi \notin \sigma^\ell$ and there is $\varphi$ such that $\varphi \cup \psi \in \sigma^\ell$. Moreover by (2), $t\left(\sigma^{\leq k}\right) \neq$ Nop for all $k > \ell$. Therefore, by (1): (i) for all $k \geq \ell$, $\sigma^k = h_{0k}$, (ii) for all $k > \ell$, $e_k \subseteq e_{\ell+1}$, and (iii) there is $\psi$ such that for all $k > \ell$, $\psi \in e_k$ and $\psi \notin \sigma^k$. But it can easily be checked that $\sigma^{\geq \ell}$ is a $\diamond$-path in $t$. Therefore $\sigma^{\geq \ell}$ is fulfilling and there must exist $k > \ell$ such that $\psi \in \sigma^k$. □

Lemma 9.3. *If a pre-model $t$ on $Cl(\varphi_0)$ is accepted by $\mathcal{A}_2$ then $t$ satisfies condition* (2) *of Def. 7.5 and $\varphi_0 \in h_0$ for $(H, \sim, h_0) = t(\epsilon)$.*

Proof. Let $t_r$ be an accepting run of $\mathcal{A}_2$ on a tree $t$, and $\sigma$ a $\diamond$-path in $t$ with initial point $\theta$. We will prove that $\sigma$ is fulfilling. Let us define, for all $k > 0$, $(c_k, e_k) \doteq t_r\left((\theta\sigma)^{\leq k}\right)$ and, since $\theta\sigma$ is active, $(H_k, \sim_k, h_{0k}) \doteq t\left((\theta\sigma)^{\leq k}\right)$. If there is $\ell$ such that $c_\ell = \mathrm{Cl}(\varphi_0)$ then $c_k = e_k = \mathrm{Cl}(\varphi_0)$ for all $k \geq \ell$ and $t_r$ would not be accepting. Therefore, for all $k$, $c_k \subseteq h_{0k}$ (in particular $\varphi_0 \in h_{00}$) and $c_{k+1} = \left((\theta\sigma)^{k+1}\right)^{\mathrm{X}}$. Hence, for all $k > 0$, $\left(\sigma^k\right)^{\mathrm{X}} \subseteq \sigma^{k+1}$ because $\sigma^{k+1} = h_{0(|\theta|+k)}$. We have proved condition (1) of Def. 7.4.

Suppose now that for some $\varphi \cup \psi \in \mathcal{L}_n$ and $\ell \in \mathbb{N}^*$, $\varphi \cup \psi \in \sigma^\ell$ and $\psi \notin \sigma^k$ for all $k \geq \ell$. It can easily be checked that for all $k \geq \ell$, $\varphi \cup \psi \in \sigma^k$. Since $t_r$ is accepting, there are $\ell_1 > |\theta| + \ell$ such that $e_{\ell_1} = \emptyset$ and $\ell_2 > \ell_1$ such that $e_{\ell_2} = \emptyset$. By definition, $h_{0\ell_1} = \sigma^{\ell_1+1-|\theta|}$. Since $\ell_1 + 1 - |\theta| > k$, $\psi \in e_{\ell_1+1}$. But for $e_{\ell_2}$ to be empty, there must exist $\ell_3$ such that $\ell_1 < \ell_3 < \ell_2$ and $\psi \in h_{0\ell_3}$ which is not possible because $\ell_3 - |\theta| > \ell$ and $\psi \notin \sigma^{\ell_3-|\theta|}$. We have proved condition (2) of Def. 7.4. □

*9.1.3 Automaton for condition* (3) *of Def. 7.5.* We first define the non-deterministic Büchi word automaton $\mathcal{A}_{3N}$ over the alphabet $\mathcal{H}_{\varphi_0} \times \left(\mathbb{L}_{\varphi_0} \cup \{\mathrm{Nop}\}\right)$. Let $\mathcal{A}_{3N} = (\mathcal{S}_{3N}, S_{03N}, \rho_{3N}, \mathcal{F}_{3N})$ with $S_{3N} \doteq 2^{\mathrm{Cl}(\varphi_0)} \times 2^{\mathrm{Cl}(\varphi_0)}$, $S_{03N} \doteq (\emptyset, \emptyset)$, $\mathcal{F}_{3N} \doteq \{(c, e) \mid e = \emptyset\}$ and $\rho_{3N}((c_k, e_k), (h, \alpha)) = (c_{k+1}, e_{k+1})$ iff one of the following condition holds:

- $\alpha = (H, \sim, h_0)$ for some $(H, \sim, h_0) \in \mathbb{L}_{\varphi_0}$, $c_k = \emptyset$, $c_{k+1} \in \mathcal{H}_{\varphi_0}$ and $e_{k+1} = \emptyset$.
- $\alpha = (H, \sim, h_0)$ for some $(H, \sim, h_0) \in \mathbb{L}_{\varphi_0}$, $c_k \in \mathcal{H}_{\varphi_0}$, $c_k$ and $h$ are $\square$-compatible, $c_{k+1} \in \mathcal{H}_{\varphi_0}$, $(c_k)^{\mathrm{X}} \subseteq c_{k+1}$ and if $e_k = \emptyset$ then $e_{k+1} = \{\psi \mid \psi \notin c_k \text{ and } \exists \varphi, \varphi \cup \psi \in c_k\}$ else $e_{k+1} = e_k \setminus c_k$.
- $c_{k+1} = e_{k+1} = \mathrm{Cl}(\varphi_0)$.
- $\alpha =$ Nop and $e_{k+1} = \emptyset$.

Given a pre-model $t$ on $\mathrm{Cl}(\varphi_0)$ and a path $\sigma$ in $t$, a sequence $\theta \in \left(\mathcal{H}_{\varphi_0} \times \left(\mathbb{L}_{\varphi_0} \cup \{\mathrm{Nop}\}\right)\right)^\omega$ *represents* $\sigma$ iff $\theta^1 = (h, t(\epsilon))$ for some $h \in \mathcal{H}_{\varphi_0}$ and for all $k > 1$, $\theta^k = \left(\sigma^{k-1}, t(\sigma^{\leq k-1})\right)$. By a reasoning similar to the proofs of Lemmas 9.2 and 9.3, the following lemma can easily be proved.

Lemma 9.4. *Let $t$ be a pre-model on $Cl(\varphi_0)$. If $t$ is a pseudo-model then all sequences representing a path in $t$ are accepted by $\mathcal{A}_{3N}$. Conversely, if all sequences representing a path in $t$ are accepted by $\mathcal{A}_{3N}$ then $t$ satisfies condition* (3) *of Def. 7.5.*

By Piterman's construction [23], $\mathcal{A}_{3N}$ can be converted into an equivalent deterministic Streett automaton $\mathcal{A}_{3D}$ on words over the alphabet $\mathcal{H}_{\varphi_0} \times \left(\mathbb{L}_{\varphi_0} \cup \{\mathrm{Nop}\}\right)$. The number of states of $\mathcal{A}_{3D}$ is double exponential in $|\varphi_0|$ and the number of its termination pairs is exponential in $|\varphi_0|$. Finally, $\mathcal{A}_{3N}$ can be converted into an equivalent deterministic Streett automaton $\mathcal{A}_3$ over pre-models by adding the label of each edge into the state of the automaton. The number of states of $\mathcal{A}_3$ is still double exponential in $|\varphi_0|$ and the number of its termination pairs is still exponential in $|\varphi_0|$.

## 9.2 Complexity

To determine the satisfiability of a formula $\varphi_0$, the automaton $\mathcal{A}$ is constructed as the product of the automata $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$. Since the number of local pseudo-models is double exponential in $|\varphi_0|$, the construction of $\mathcal{A}$ takes double exponential time. The number of states of $\mathcal{A}$ is double exponential in $|\varphi_0|$ and the number of its termination pairs is exponential in $|\varphi_0|$. Emerson and Jutla [10] proved that the emptiness of a Streett tree automaton with $s$ states and $p$ termination pairs can be decided in $(s \cdot p)^{O(p)}$ deterministic time. Hence, the emptiness problem for $\mathcal{A}$ can be decided in double exponential time in $|\varphi_0|$. By combining the lemmas of the previous sections we have that $\mathcal{A}$ is empty if and only if $\varphi$ is unsatisfiable. Since CTL* is 2EXPTIME-hard [30] and can be faithfully translated into DT-STIT$_n$, we have proved the following theorem.

Theorem 9.5. *The satisfiability problem of DT-STIT$_n$ is 2EXPTIME-complete.*

## 10 CONCLUSION

We have provided a new semantics based on concurrent game structures (CGSs) for a temporal STIT logic that extends CTL* by agency operators. We have proved that the semantics based on CGSs and the semantics based on discrete *BT+AC* structures are equivalent for this logic. Furthermore, we have proved that the satisfiability problem of the individual STIT fragment of our logic is 2EXPTIME-complete, the same complexity as CTL*.

In future work, we plan to introduce a more practical semantics for our temporal STIT logic based on the model representation known as "simple reactive modules language" (SRML) [28], a simplified version of "reactive modules language" (RML) by [1], used in model checkers such as SMV and MOCHA. SRML describes models in a more concise way than CGSs. As shown by [28], satisfiability checking and model checking for ATL are both EXPTIME-complete when using SMRL. On the contrary, when using CGSs, satisfiability checking for ATL is EXPTIME-complete [29, 31], while model checking is solvable in polynomial time [2, 13]. We will verify whether the same kind of phenomenon appears in the context of our temporal STIT logic, namely whether complexity of satisfiability checking for our logic does not increase when moving from CGSs to SMRL, while complexity of model checking does.

We also plan to study an epistemic extension of our temporal STIT logic, after having enriched CGSs with epistemic accessibility relations for representing agents' uncertainties. We expect this epistemic extension of our logic to be well-suited to model repeated games with imperfect information.

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