Incentives to Invite Others to Form Larger Coalitions

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ABSTRACT

We study a cooperative game setting where players form a network and each player only knows the existence of the players to whom she connects. Initially, only a subset of the players are in the game. Our goal is to design a reward distribution mechanism to incentivize the players to use their connections to invite more players to join the game. We show that the existing solutions such as the Shapley value cannot achieve this. Hence, to combat this problem, we propose a solution called *weighted permission Shapley value* (inspired by permission structure and the weighted Shapley value). Under this solution, for each player, inviting all her neighbors is a dominant strategy in all monotone games. We further prove that the solution is unique for tree networks. Our solution offers the very first attempt to incentivize the players to invite others to form a larger coalition in cooperative games.

KEYWORDS

Mechanism design, Cooperative games, Diffusion incentives

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1 INTRODUCTION

Cooperative game is a classical research topic studied in game theory [3, 11, 20]. The study focused on games where a fixed set of players forms coalitions to share rewards. One goal is to design a reward distribution to incentivize all players to collaborate in the grand coalition. The literature assumed that all the players are fixed and there is no connection between players. However, in reality, the players are connected and they could potentially use their connections to involve more players or exclude some players.

In this paper, we aim to design a reward distribution mechanism to incentivize the participants who are already in the game to invite others to join the collaboration. This kind of mechanism is highly demanded in practice. For instance, when we collect a large scale data-set from the crowd, select a set of seed users for a product trial, or look for a missing person, we could use social networks to attract participants [6, 15]. Among them, the most successful example is the well-known network challenge hosted by DARPA in 2009 where a collaboration formed via social networks played an essential role [12].

Different from the traditional settings, players in our setting are connected and a player cannot join the collaboration without the invitation from her neighbours (simply because the player is not aware of the collaboration without the others' invitation). *More precisely, our challenge is to design a reward distribution mechanism such that the current players of the coalition are incentivized to invite their neighbours to join the coalition.*

The Shapley value is a classic solution concept in cooperative games to distribute rewards [14]. It computes the average marginal contribution of each player to join a group. However, in the calculation, all the players are treated equally and each of them can join any group, which is not the case in our setting because if one player is invited by another, they cannot be treated equally. Therefore, it is easy to show that the Shapley value cannot be directly applied here to incentivize the players to invite others.

To tackle this problem, we propose a solution based on the concepts of permission structure and the weighted Shapley value. The weighted Shapley value is the very first concept that applies asymmetry to cooperative games [1, 7, 13]. However, the asymmetry induced by the weights cannot reflect the structure of the invitations among the players because the weight of each player is the same for all coalitions. Another concept called permission structure seems a closer solution to our problem, which was first introduced in [4, 5] and characterized in [17, 18]. In a permission structure, players need permissions from other players before they are allowed to cooperate, where the permissions are very similar to the players' invitations in our model. In our model, a player is not aware of the game without someone's invitation, but permission structure does not offer the flexibility on the reward distribution.

Against this background, our solution is a novel combination of the weighted Shapley value and the permission structure. We use a permission structure to represent the priorities between an inviter and an invitee, and assign different weights to them to control the importance of their priorities. For the first time, we show that the well-known winning solution for the DARPA network challenge is a special case of our solution [12]. Our solution will stimulate a broad application of collaborations via social networks such as crowdsourcing and question answering. Our contributions advance the state of the art in the following ways:

- We formally model the (social) connections between players in a cooperative game and, for the first time, define the concept of *diffusion incentive compatibility (DIC)* for players to utilize their connections to gather more players.
- We define a *weighted permission Shapley value* as a reward distribution mechanism to achieve DIC.
- We also formally model the query network as a special case of our setting and show that our solution is the only solution to achieve DIC in the query network.

The key contribution of our work is to introduce diffusion incentives to cooperative games for the first time, which utilizes the connections between the players to invite more players to the game.

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Utilizing the power of social network to involve more participants is a new trend in mechanism design, especially in non-cooperative games [16, 21]. For instance, in auctions, Li *et al.* [9, 10] and Zhao *et al.* [22] proposed the very first diffusion mechanisms to attract buyers via social networks. Our model shares a similar motivation, but it cannot be handled with their techniques because they focused on the non-cooperative games and the participants' contribution also depends on their private valuations.

The remainder of the paper is organized as follows. Section 2 gives a formal description of the model. Section 3 establishes the family of weighted permission Shapley value to incentivize diffusion in forests and Section 4 demonstrates its applicability in query networks. Section 5 extends the result to general graphs.

2 THE MODEL

We study a cooperative game where players are connected to form a network and each player only knows the players she connects to. In real-world applications, their connections can represent friendship or leadership. Initially, only a subset of the players are aware of the game. A person who is in the game can invite her friends who are not in the game yet to join. We investigate the reward distribution mechanism in this setting to incentivize the existing players to invite new players to join the coalition.

Formally, let $N = \{1, 2, \ldots, n\}$ be the set of all connected players in the underlying network. We model the network as a directed acyclic graph (DAG) $G = (N, E)$[1]. Each edge $e = (x, y) \in E$ indicates that player x can invite y. There is a special player set $\mathcal{I} \subseteq N$ who are in the game/coalition initially without invitation. We call $\mathcal{I}$ *the initial set* and the invitation has to start from the initial players. For each player $i \in N$, let $p_i = \{j \mid (j, i) \in E\}$. We have $p_i = \emptyset$ if and only if $i \in \mathcal{I}$. We may assume w.l.o.g. that all players can be reached from at least one of the initial players in the underlying network. Let $\theta_i = \{j \mid (i, j) \in E\}$ be private type of player i, which is the set of players who can be invited by i. Let $\theta = (\theta_1, \ldots, \theta_n)$ be the type profile of all players, and θ_{-i} be the type profile of the players except for i. Let Θ_i, Θ and Θ_{-i} be the space of θ_i, θ and θ_{-i} respectively.

As the network is not public, the reward distribution mechanism needs the players' report about their connections (types) to involve all players. Let $\theta'_i \subseteq \theta_i$ is the type report of player i, i.e., the actual player set invited by i. Given any report profile $\theta' = (\theta'_1, \ldots, \theta'_n)$, there is a directed graph induced by θ', denoted by $G(\theta') = (N, E(\theta'))$, where $E(\theta') = \{(i, j) \mid i \in N, j \in \theta'_i\}$. Let $J_{\mathcal{I}}(G(\theta'))$ be the set of players who can be reached from at least one player from $\mathcal{I}$ under $G(\theta')$. It is clear that only the players in $J_{\mathcal{I}}(G(\theta'))$ can actually join the game, because the others cannot receive the proper invitation started from the initial players (in practice, this means that the others will not be informed about the game at all).

There is a non-negative and monotone characteristic function $v : 2^N \to \mathbb{R}$, s.t., $v(\emptyset) = 0$ and $v(S) \le v(T)$ for all $S \subseteq T \subseteq N$. The monotone property is necessary in our setting; otherwise, there is no need to invite more people to join the coalition if fewer people can do better. Note that the definition of v does not consider the connections between players, simply because the connections are their private information, which is what we need to discover. Let $\mathcal{V}$ be the space of all monotone characteristic functions for N. We define the reward distribution mechanism as follows.

Definition 2.1. A **reward distribution mechanism** $\mathcal{M}$ is defined by a reward policy $\Phi = \{\phi_i\}_{i \in N}$, where each $\phi_i : \Theta \times \mathcal{V} \to \mathbb{R}_{\ge 0}$ assigns the reward to player $i \in N$. Moreover, for all $\theta' \in \Theta$ and all $v \in \mathcal{V}$, $\phi_i(\theta', v) = 0$ if $i \notin J_{\mathcal{I}}(G(\theta'))$.

The mechanism does not distribute reward to players who are not invited. Except for this, one desirable property of the mechanism is to distribute exactly what the coalition (of all participated players) can generate. This is called *efficiency*.

Definition 2.2. A reward distribution mechanism $\mathcal{M}$ is **efficient** if for all $\theta' \in \Theta$ and all $v \in \mathcal{V}$, we have

$$\sum_{i \in N} \phi_i(\theta', v) = v\left(J_{\mathcal{I}}(G(\theta'))\right)$$

Other than efficiency, the key property we want to achieve here is to incentivize all players who are already in the coalition to invite all their neighbours to join the coalition. This requires that inviting all neighbours is a dominant strategy for all players. We call this property *diffusion incentive compatibility.*

Definition 2.3. A reward distribution mechanism $\mathcal{M}$ is **diffusion incentive compatible (DIC)** if for all $i \in N$ with type $\theta_i \in \Theta_i$, all $\theta'_i \subseteq \theta_i$, all $\theta'_{-i} \in \Theta_{-i}$ and all $v \in \mathcal{V}$, we have

$$\phi_i((\theta_i, \theta'_{-i}), v) \ge \phi_i((\theta'_i, \theta'_{-i}), v)$$

Finally, we consider a property of structural fairness that guarantees a player can gain a reward at least as much as a fixed proportion of the reward gained by her invitees.

Definition 2.4. A reward distribution mechanism $\mathcal{M}$ has the property of γ-**structural fairness (**γ**-SF)** if for all $v \in \mathcal{V}$, all $\theta' \in \Theta$ and all $i, j \in J_{\mathcal{I}}(G(\theta'))$ with $j \in \theta'_i$, we have $\phi_i(\theta', v) \ge \gamma \phi_j(\theta', v)$.

2.1 Shapley Value does not Work

Before proposing our solution, the first question is what if we directly apply the Shapley value [14, 19], the classical solution for cooperative games, in our setting.

Let $\mathcal{R}(S)$ denote the set of all orders R of players in a coalition S. For an order R in $\mathcal{R}(N)$, we use $B^{R,i}$ to denote the set of players preceding i in the order R. For a given characteristic function v and an order R, the marginal contribution of player i in R is $C_i(v, R) = v(B^{R,i} \cup \{i\}) - v(B^{R,i})$. Then the classical Shapley value of game v for i is the expectation of i's marginal contribution:

$$\varphi_i(v) = \mathbb{E}_{\mathcal{U}_{\mathcal{R}(N)}}(C_i(v, \cdot))$$

where $\mathcal{U}_{\mathcal{R}(N)}$ is a uniform distribution on $\mathcal{R}(N)$. We show that the Shapley value (on $J_{\mathcal{I}}(G(\theta'))$) does not work in our new setting.

Proposition 2.5. *If the Shapley value is applied as a reward distribution mechanism $\mathcal{M}$, then $\mathcal{M}$ is not diffusion incentive compatible.*

Proof. We prove this by a counterexample. Consider the game in Example 2.6. When applying the Shapley value as the reward distribution mechanism, we get

$$\varphi_1 = \varphi_3 = 1/2, \quad \varphi_2 = 0, \quad \varphi_4 = 1.$$

[1] Our results are not restricted to DAGs (see Section 5.3 for the extension to general graphs).

Notice that agents 1 and 3 have the same contribution to other agents and they will share the reward of that contribution. However, if agent 1 does not invite agent 3, her Shapley value will be increased to $1 > 1/2$. Hence, directly applying the Shapley value is not diffusion incentive compatible. □

Example 2.6. Consider the case illustrated in Figure 1, where $N = \{1, 2, 3, 4\}$. Suppose the initial coalition is $\mathcal{I} = \{1, 2\}$. To form a larger coalition, agents 1 and 2 can invite their friends 3 and 4. Suppose the characteristic function v is defined as

$$v(S) = \mathbb{I}(\{1, 3\} \cap S \neq \emptyset) + \mathbb{I}(4 \in S)$$

for all $S \subseteq N$, where $\mathbb{I}(\cdot)$ is the indicator function.

Figure 1: An example of the cooperative game in a forest.

3 DIFFUSION INCENTIVES IN A FOREST

In this section, we first investigate the solution to satisfy efficiency and DIC when the network G is a forest. We will continue using Example 2.6 to illustrate our notions and results.

3.1 Permission Structure in Forests

To tackle the failure of Shapley value, we recall an important concept called permission structure. Gilles et al. [4, 5] firstly gave the permission restriction on cooperative games. It restricts a player's impact by a permission group. We will use it here to represent the invitations. Intuitively, a permission structure can represent how players get involved in the game by others' invitations/permissions.

Definition 3.1. A permission structure on N is an asymmetric mapping $P : N \to 2^N$, i.e., $j \in P(i)$ implies that $i \notin P(j)$.

Here, we define $P(i)$ as the set of players who invited i into the coalition, i.e., $P(i) = p'_i = \{j \mid i \in \theta'_j\}$. In particular, in the forest model, every player except for the initial players has a unique parent who invites her ($|p'_i| = 1$). For instance, in Example 2.6, $P(3) = \{1\}$ and $P(4) = \{2\}$. With permission structure, we can define the autonomous coalition.

Definition 3.2. A coalition $S \subseteq N$ is autonomous in a permission structure P if for all $i \in S$, $P(i) \subseteq S$.

A coalition S is autonomous if and only if for each player $i \in S$, all her ancestors are also in S. This property indicates whether a coalition is able to collaborate together (generate a reward). For instance, in Example 2.6, $\{1, 3\}$ is autonomous while $\{4\}$ is not autonomous. Denote the collection of all autonomous coalitions in permission structure P by A_P. Then for an arbitrary coalition $S \subseteq N$, we consider the largest autonomous coalition within S.

Definition 3.3. Given a permission structure P on N, the largest autonomous coalition of $S \subseteq N$ is defined as

$$\alpha(S) = \bigcup\{T \mid T \subseteq S \text{ and } T \in A_P\}.$$

Intuitively, $\alpha(S)$ is the largest subset of S that is autonomous. In particular, in the forest model, let G_S be the subgraph of the forest G formed by players in S, then the largest autonomous coalition of a coalition S is all the connected components of G_S where each component contains at least one player in the initial set $\mathcal{I}$. For instance, in Example 2.6, the largest autonomous coalition of set $\{1, 3, 4\}$ is $\{1, 3\}$.

3.2 Applying Permission Shapley Value

Considering the diffusion network, we see that some players need others' participation/invitation to create value. For instance, in Example 2.6, player 4 can only be invited by player 2. Hence, without player 2, player 4 cannot join the game to provide her contribution to the coalition. This suggests applying Shapley value with a permission structure.

Taking the notations in Section 3.1, with the restriction of the permission structure, we can map the characteristic function v of the cooperative game to a projection v^P on P as

$$v^P(S) = v(\alpha(S))$$

for all $S \subseteq N$ [5]. $v^P(S)$ defines the contribution of a coalition S by the contribution of those players who can actually participate under the coalition S.

Define the Shapley value under permission structure P on game v as $\varphi^P(v) = \varphi(v^P)$, i.e., the Shapley value on game v^P. Let's call it *permission Shapley value*. Applying the permission Shapley value in Example 2.6, we have

$$v^P(S) = \mathbb{I}(1 \in S) + \mathbb{I}(\{2, 4\} \subseteq S)$$

and the reward distribution is

$$\varphi^P_1 = 1, \quad \varphi^P_3 = 0, \qquad \varphi^P_2 = \varphi^P_4 = 1/2.$$

From the example we can see two intuitions of the permission Shapley value. Firstly, if a player has the same contribution as her inviter (e.g., player 3 and her inviter player 1 in Example 2.6), then only the inviter will be rewarded for that contribution. This can be naturally obtained from the property of diffusion incentive compatibility. No matter how much reward the player shared with her inviter, the inviter will then have no incentives to invite the player. On the other hand, if a player invites another player who can bring additional contribution (e.g., player 2 and player 4 in Example 2.6), then the reward of invitee will be equally shared among the inviter and the invitee.

To see the intuition, we consider a special case where an additive assumption is applied, i.e., for each two disjoint subsets of players $S_1, S_2 \subseteq N$, we have $v(S_1 \cup S_2) = v(S_1) + v(S_2) \geq 0$. Under this assumption, denote the depth of i in the tree i belongs to by d_i (the depths of the roots are 0) and the subtree rooted by i by T_i. Then the permission Shapley value of all player $i \in N$ is

$$\varphi^P_i = \sum_{k \in T_i} \frac{v(\{k\})}{d_k + 1} \qquad \text{for every } i \in N.$$

Intuitively speaking, the contribution of a player will be uniformly distributed along the invitation chain.

Now, we show that the permission Shapley value is a desirable reward distribution mechanism that satisfies efficiency and diffusion incentive compatibility for the cooperative game in forests.

THEOREM 3.4. *For the monotone diffusion cooperative game in a forest, if the reward distribution mechanism $\mathcal{M}$ is the permission Shapley value, then $\mathcal{M}$ is efficient and DIC.*

PROOF. (i) $\mathcal{M}$ is efficient since for all $\theta' \in \Theta$

$$\sum_{i \in N} \varphi_i^P(v) = \sum_{i \in N} \varphi_i(v^P) = v^P(J_{\mathcal{I}}(G(\theta'))) = v(J_{\mathcal{I}}(G(\theta'))).$$

(ii) For the diffusion incentive compatibility, we will show that for each player i, her permission Shapley value is non-decreasing after she invites more players in the game. Consider a player set X which cannot be informed of the game if i does not invite some players. Let P be the permission structure if i does not invite these players and P' be the one if i invites these players. Then,

- before i invites some players to let X get involved in the game, the permission Shapley value of i is

$$\varphi_i^P(v) = \varphi_i(v^P) = \frac{1}{|N \setminus X|!} \sum_{R \in \mathcal{R}(N \setminus X)} C_i(v^P, R);$$

- if i invites players such that X then can be involved in the game, notice that for all $R \in \mathcal{R}(N \setminus X)$ and all $Y \subseteq X$, we have $v^P(B^{R,i}) = v^{P'}(B^{R,i} \cup Y)$ since i is not in the coalition. Denote $C_Y(v^{P'}, R, i) = v^{P'}(B^{R,i} \cup Y \cup \{i\}) - v^{P'}(B^{R,i} \cup \{i\})$. Then the permission Shapley value of i will become

$$\begin{aligned}
\varphi_i^{P'}(v) &= \frac{1}{|N|!} \sum_{R \in \mathcal{R}(N)} C_i(v^{P'}, R) \\
&= \frac{|X|!}{|N|!} \sum_{R \in \mathcal{R}(N \setminus X)} \left[\binom{|N|}{|X|} C_i(v^P, R) + \sum_{Y \subseteq X} C_Y(v^{P'}, R, i) \right] \\
&\geq \frac{|X|!}{|N|!} \sum_{R \in \mathcal{R}(N \setminus X)} \binom{|N|}{|X|} C_i(v^P, R) \\
&= \frac{1}{(|N| - |X|)!} \sum_{R \in \mathcal{R}(N \setminus X)} C_i(v^P, R) = \varphi_i^P(v)
\end{aligned}$$

where the equality in the second line means the marginal contribution gained by i when asserting the players in X to all orders in $\mathcal{R}(N \setminus X)$. Therefore, the permission Shapley value is DIC. □

For structural fairness, we show that permission Shapley value satisfies 1-SF. Intuitively, it means that if a player i invites j to the game, then she will gain at least as much as the reward that is distributed to j.

THEOREM 3.5. *For the monotone diffusion cooperative game in a forest, if the reward distribution mechanism $\mathcal{M}$ is the permission Shapley value, then $\mathcal{M}$ is 1-SF.*

PROOF. For all $\theta' \in \Theta$, consider $i, j \in J_{\mathcal{I}}(G(\theta'))$ with $j \in \theta'_i$. For all $S \subseteq N$ with $i \in S$, let $Q_S^i = (|N| - |S|)!(|S| - 1)!/|N|!$, i.e., the probability of $B^{R,i} = S \setminus \{i\}$ in all orders $R \in \mathcal{R}(N)$. Since R is sampled with uniform distribution, then for all $S \subseteq N$ with $i, j \in N$, $Q_S^i = Q_S^j$. Hence, we have

$$\begin{aligned}
\varphi_i^P &= \sum_{S \ni i} Q_S^i \left[v^P(S) - v^P(S \setminus \{i\}) \right] \\
&= \sum_{S \ni i, S \not\ni j} Q_S^i \left[v^P(S) - v^P(S \setminus \{i\}) \right] \\
&\quad + \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{i\}) \right] + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot 0 \\
&\geq \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{i\}) \right] + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot 0 && (1) \\
&= \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{i\}) \right] \\
&\quad + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot \left[v^P(S) - v^P(S \setminus \{j\}) \right] && (2) \\
&\geq \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{j\}) \right] \\
&\quad + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot \left[v^P(S) - v^P(S \setminus \{j\}) \right] && (3) \\
&= \sum_{S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{j\}) \right] = \varphi_j^P
\end{aligned}$$

where the Inequality (1) is satisfied since the game is monotone; the Equality (2) is satisfied since for all $S \subseteq N$ with $i \notin S$ and $j \in S$, $j \notin \alpha(S)$ and then $v^P(S) - v^P(S \setminus \{j\}) = v(\alpha(S)) - v(\alpha(S)) = 0$; the Inequality (3) is satisfied since for all $S \subseteq N$ with $i, j \in S$, $\alpha(S \setminus \{i\}) = \alpha(S \setminus \{i, j\})$ and then $v^P(S) - v^P(S \setminus \{i\}) = v^P(S) - v^P(S \setminus \{i, j\}) \geq v^P(S) - v^P(S \setminus \{j\})$. □

3.3 Using Weights to Further Utilize the Structure

In the game of Example 2.6, the permission Shapley value suggests an equal share between players 2 and 4 for 4's contribution. However, in this example, player 2 will have diffusion incentives if we give any fraction of player 4's contribution to him. That means we can further tune the above mechanism to have the same properties. One way to tune the share between an inviter and an invitee is to introduce weights to them.

Kalai and Samet [7] introduced weights to the Shapley value as an alternative solution to cooperative games. Radzaik [13] further discussed the variants and properties of the weighted Shapley value and Dragan [2] provided a computation method for weighted Shapley value. Usually, the weighted Shapley value can be defined as:

$$\varphi_i^\omega(v) = \mathbb{E}_{\mathcal{D}(\omega)}(C_i(v, \cdot))$$

where $\omega = (\omega(1), \omega(2), \ldots, \omega(n)) \in \mathbb{R}_+^N$ are the weights assigned to players and $\mathcal{D}(\omega)$ is a distribution on $\mathcal{R}(N)$ based on ω.

To compute $\mathcal{D}(\omega)$, consider an order $R = (i_1, i_2, \ldots, i_n) \in \mathcal{R}(N)$, define $\omega_R = \prod_{k=1}^m \left(\omega(i_k) / \sum_{p=1}^k \omega(i_p) \right)$. This can be interpreted as the probability of sampling the order R by agents' weights, e.g., sampling last player as i_n has probability $\omega(i_n)/(\omega(i_1) + \cdots + \omega(i_n))$ and sampling previous player as i_{n-1} in the remaining players has probability $\omega(i_{n-1})/(\omega(i_1) + \cdots + \omega(i_{n-1}))$. Finally, in $\mathcal{D}(\omega)$, the probability of selecting order R is ω_R [7]. Table 1 shows an example of the weight assignments.

order R	ω_R ($\omega(i) = 1$)	ω_R ($\omega(1 \text{ or } 2) = 1, \omega(3) = 2$)
(1, 2, 3), (2, 1, 3)	1/6	1/4
(1, 3, 2), (2, 3, 1)	1/6	1/6
(3, 1, 2), (3, 2, 1)	1/6	1/12

Table 1: An example of computing the $\mathcal{D}(\omega)$ from weights ω. We can see when $\omega(3)$ is larger, player 3 has more chance to appear the the later positions.

Note that when $\omega = 1^{|N|}$, the weighted Shapley value becomes the classical Shapley value. In our setting, we can also assign weights to players. Intuitively, the permission structure shows some kinds of “external" relations of the players: how players are connected; while the weights show some “internal" relations: which player takes a more important role in a coalition. Thus, these two solution concepts are of different classes. In our reward distribution mechanism, we want to consider not only the “external" structures but also “internal" relations between the players involved. Moreover, from the perspective of fairness, the weights will decide how much a player i can be rewarded by inviting her neighbours, i.e., the parameter γ in structural fairness. Thus, we extend the permission Shapley value by adding weights as weighted permission Shapley value.

Definition 3.6. For a cooperative game v on the player set N, given a permission structure P and weights $\omega \in \mathbb{R}_+^{|N|}$, the weighted permission Shapley value for a player $i \in N$ is:

$$\varphi_i^{\omega,P}(v) = \varphi_i^{\omega}(v^P).$$

To apply the weighted permission Shapley value to our cooperative game, we need to define the weight function $\omega(i)$ to set the weights to each player i. As an example, let the weight function be $\omega(i) = d_i + 1$, where d_i is the depth of player i in the tree i belongs to[2]. Then applying the weighted permission Shapley value in Example 2.6, the rewards distributed to players are:

$$\begin{cases} \varphi_1^{\omega,P} = 1, & \varphi_3^{\omega,P} = 0, \\ \varphi_2^{\omega,P} = 1/3, & \varphi_4^{\omega,P} = 2/3. \end{cases}$$

From the example we can see that we make a difference between players 2 and 3's rewards. Again, consider the special case for the additive characteristic function v. Let T_i be the subtree rooted by i. Then the weighted permission Shapley value with weight $\omega : \omega(i) = f(d_i)$ for all player $i \in N$ is

$$\varphi_i^{\omega,P} = \sum_{k \in T_i} \frac{f(d_i)}{\sum_{j=0}^{d_k} f(j)} v(\{k\}).$$

Intuitively speaking, the reward will be distributed along the invitation chain according to the ratio of the weights rather than uniformly divided.

If we set weights as $1^{|N|}$, the weighted permission Shapley value will become normal permission Shapley value. Hence, the weighted permission Shapley value is a more general class. We will show that if we set weights properly, it is also a desirable solution to satisfy efficiency and diffusion incentive compatibility.

Theorem 3.7. *For the monotone diffusion cooperative game in a forest, if the reward distribution mechanism $\mathcal{M}$ is the weighted permission Shapley value with weight function $\omega(i) = f(d_i)$ for all player i, which only depends on her distance to initial players, then $\mathcal{M}$ is efficient and DIC.*

Proof. (i) $\mathcal{M}$ is efficient since for all $\theta' \in \Theta$,

$$\sum_i \varphi_i^{\omega,P}(v) = \sum_i \varphi_i^{\omega}(v^P) = v^P(J_{\mathcal{I}}(G(\theta'))) = v(J_{\mathcal{I}}(G(\theta'))).$$

(ii) For the property of DIC, suppose X is the player set which cannot be informed of the game if i does not invite some players. Let P be the permission structure if i does not invite these players and P' be the permission structure if i invites these players. Then, the weighted permission Shapley value before i let X get involved in the game is

$$\varphi_i^{\omega}(v^P) = \sum_{R \in \mathcal{R}(N \setminus X)} \omega_R C_i(v^P, R) \Bigg/ \sum_{R \in \mathcal{R}(N \setminus X)} \omega_R .$$

Consider an order $R_j^p = (i_1, \dots, i_{p-1}, j, i_p, \dots, i_m)$, which inserts player j at the position p in R. Then from the definition we can derive that $\omega_R = \sum_{p=1}^{m+1} \omega_{R_j^p}$ if for all k, $\omega(i_k)$ will not change after j joins in. More generally, for any additional player set X, if for all k, $\omega(i_k)$ will not change after X joins in, we have $\sum_{R' \in R_X} \omega_{R'} = \omega_R$, where R_X is the set of all possible orders that insert all players in X into the order R. Then if i invites players such that X can be involved in the game, since the weight function $\omega(i) = f(d_i)$ only depends on d_i, for all player $i \in N \setminus X$, $\omega(i)$ will not change. Hence, the weighted permission Shapley value of i becomes

$$\begin{aligned} \varphi_i^{\omega}(v^{P'}) &= \frac{1}{\sum_{R \in \mathcal{R}(N)} \omega_R} \sum_{R \in \mathcal{R}(N)} \omega_R C_i(v^{P'}, R) \\ &= \frac{1}{\sum_{R \in \mathcal{R}(N \setminus X)} \omega_R} \sum_{R \in \mathcal{R}(N \setminus X)} \sum_{R' \in R_X} \omega_{R'} C_i(v^P, R') \\ &\geq \frac{1}{\sum_{R \in \mathcal{R}(N \setminus X)} \omega_R} \sum_{R \in \mathcal{R}(N \setminus X)} \sum_{R' \in R_X} \omega_{R'} C_i(v^P, R) \\ &= \frac{1}{\sum_{R \in \mathcal{R}(N \setminus X)} \omega_R} \sum_{R \in \mathcal{R}(N \setminus X)} \omega_R C_i(v^P, R) = \varphi_i^{\omega}(v^P). \end{aligned}$$

Therefore, $\mathcal{M}$ is DIC.

□

Moreover, by introducing weights, we can make the structural fairness more tunable to customize the requirements in different scenes.

Theorem 3.8. *For the monotone diffusion cooperative game in a forest, if the reward distribution mechanism $\mathcal{M}$ is the weighted permission Shapley value with weight function $\omega(i)$ that satisfies for all $\theta' \in \Theta$ and for all $i, j \in J_{\mathcal{I}}(G(\theta'))$ with $j \in \theta_i'$, $\omega(i)/\omega(j) \geq \gamma$, then $\mathcal{M}$ is γ-SF.*

[2] For players who are not in the set $J_{\mathcal{I}}(G(\theta'))$, they can be assigned arbitrary positive weight. We will not specify the conditions for these players in the rest unless necessary.

Proof. For all $\theta' \in \Theta$, consider $i, j \in J_{\mathcal{I}}(G(\theta'))$ with $j \in \theta'_i$. For all $S \subseteq N$ with $i \in S$, let Q_S^i be the probability of $B^{R,i} = S \setminus \{i\}$ in all orders $R \in \mathcal{R}(N)$. Since R is sampled with distribution $\mathcal{D}(\omega)$, then for all $S \subseteq N$ with $i, j \in N$, we have

$$\frac{Q_S^i}{Q_S^j} = \frac{\omega(i)}{\omega(j)}.$$

Hence, $Q_S^i = \frac{\omega(i)}{\omega(j)} Q_S^j$ and we have

$$\begin{aligned}
\varphi_i^{\omega,P} &= \sum_{S \ni i} Q_S^i \left[v^P(S) - v^P(S \setminus \{i\}) \right] \\
&= \sum_{S \ni i, S \not\ni j} Q_S^i \left[v^P(S) - v^P(S \setminus \{i\}) \right] \\
&\quad + \frac{\omega(i)}{\omega(j)} \left\{ \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{i\}) \right] + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot 0 \right\} \\
&\geq \frac{\omega(i)}{\omega(j)} \left\{ \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{i\}) \right] + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot 0 \right\} \\
&\geq \frac{\omega(i)}{\omega(j)} \left\{ \sum_{S \ni i, S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{j\}) \right] \right. \\
&\quad \left. + \sum_{S \not\ni i, S \ni j} Q_S^j \cdot \left[v^P(S) - v^P(S \setminus \{j\}) \right] \right\} \qquad (4) \\
&= \frac{\omega(i)}{\omega(j)} \left\{ \sum_{S \ni j} Q_S^j \left[v^P(S) - v^P(S \setminus \{j\}) \right] \right\} \\
&= \frac{\omega(i)}{\omega(j)} \varphi_j^{\omega,P} \geq \gamma \varphi_j^{\omega,P}
\end{aligned}$$

where the Inequality (4) is satisfied according to the same reason for Equality (2) and Inequality (3) in Theorem 3.5.

Therefore, the $\mathcal{M}$ is γ-SF. □

Intuitively, the parameter of the structural fairness is determined by $\min_{j \in \theta'_i} \omega(i)/\omega(j)$. For example, if $\omega(i) = d_i + 1$, then the corresponding weighted permission Shapley value is 1/2-SF.

4 THE ONLY SOLUTION TO QUERY NETWORK

A classic problem that can be modelled as a diffusion cooperative game is the query incentive network [8], where a requester tries to find an answer to a specific problem by diffusing the request in the network. A solution is given by the winning team from MIT in the DARPA network challenge [12]. In the challenge, each team needed to find positions of the red balloons to obtain rewards. The solution proposed by the winning team is that they promised half of the reward for the first person who found it and one-fourth of the reward to the person who invited the finder and so on. The requester (initial players) will get the remaining. An example is shown in Figure 2.

We can model it as a diffusion cooperative game with an additive characteristic function where only one agent (the answer holder) can contribute the utility. Without loss of generality, we assume there is only one initial player as the requester and only one player can provide the answer and the answer will bring one unit value (for the game in the example shown in Figure 2, we can seperate it as two games and add the two solutions up). More precisely, in the corresponding diffusion cooperative game to the query network, we set $\mathcal{I} = \{1\}$ and for any $S \subseteq N, v(S) = 1$ if and only if the answer holder $j \in S$. In general, a solution to the query network is a reward distribution $x(i)$ for all players i along the path from the requester to answer provider. We require that the reward distribution $x(i)$ satisfies the following properties.

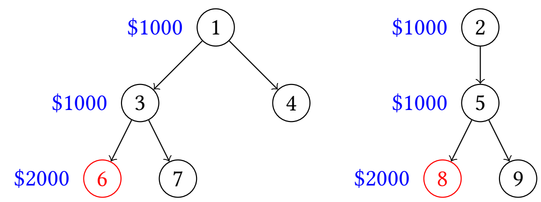

Figure 2: An example of the solution given by winning team in DARPA challenge. Players 1 and 2 are the initial team members. Players 6 and 8 are those who find the balloon.

Definition 4.1. A reward distribution $x(i)$ for all players i along the path from the requester 1 to answer provider j in the query network is

- *anonymous* if $x(i)$ only depends on d_i and d_j (the distances from player 1 to i and j, which indicates i's position);
- *strongly individually rational* (SIR) if $x(i) > 0$ for all i from 1 to j;
- *efficient* if $\sum_i x(i) = 1$.

For example, the solution given by the DARPA winning team can be described as $x(i) = 1/2^{d_j+1-d_i}$ with $i > 1$ and $x(1) = 1/2^{d_j}$ for all i on the path from 1 to j. We show that all the solution concepts can be mapped to a set of weighted permission Shapley values. In other words, the set of weighted permission Shapley value is the only satifiable solution to the query network.

Theorem 4.2. *A solution to a query network is anonymous, strongly individually rational and efficient if and only if it is a weighted permission Shapley value with $\omega(i) = f(d_i, d_j)$, where j is the answer provider.*

Proof. "⇒": suppose $x(i)$ is an anonymous, SIR and efficient solution to the query network. Construct a weighted permission Shapley value with $\omega(i) = x(i)$ for all i on the path from agent 1 to j and $\omega(i) = 1$ for other players. Then,

$$\varphi_i^{\omega,P} = \sum_{k \in T_i} \frac{\omega(i)}{\sum_{l=0}^{d_k} \omega(l)} v(\{k\}) = \frac{x(i)}{\sum_l x(l)} = x(i)$$

for all i on the path from agent 1 to j and otherwise $\varphi_i^{\omega,P} = 0$.

"⇐": consider a weighted permission Shapley value with $\omega(i) = f(d_i, d_j)$. $\varphi_i^{\omega,P} = 0$ if i is not an ancestor of j. For all i on the path from 1 to j, we have

$$\varphi_i^{\omega,P} = \frac{f(d_i, d_j)}{\sum_{k \in \text{ path from agent 1 to } j} f(d_k, d_j)} = \frac{f(d_i, d_j)}{\sum_{k=0}^{d_j} f(k, d_j)} > 0$$

which only depends on d_i and d_j. Finally, the efficiency holds since $v(N) = 1$. □

Again, take the solution of DARPA winning team as an example. Consider the weighted permission Shapley value with $\omega(i) = \max\{1, 2^{d_i-1}\}$, and we have

$$\varphi_i^{\omega,P} = \frac{2^{d_i-1}}{1 + \sum_{k=1}^{d_j} 2^{k-1}} = \frac{2^{d_i-1}}{2^{d_j}} = 1/2^{d_j+1-d_i}$$

for all i on the path from agent 1 to j with $i > 1$, $\varphi_1^{\omega,P} = 1/2^{d_j}$ and $\varphi_i^{\omega,P} = 0$ otherwise, which is equivalent to the solution of DARPA winning team. Moreover, in this example, $\omega(i)$ only depends on d_i, so that we know the solution of DARPA winning team is diffusion incentive compatible according to Theorem 3.7.

5 FROM FOREST TO DAG

In this section, we extend our result in the setting of forest to a general DAG model. An instance of a cooperative game in a DAG is shown in Example 5.1 below.

Example 5.1. Consider the case illustrated in Figure 3, where $\mathcal{I} = \{1, 2\}$. Agent 1 asks her friends 3 and 5, and 2 asks 4 and 5. Then 3, 4 and 5 further ask their friends and so on. Suppose the player 5 will join in if 2 invites her or 1 and 3 both invite her and the player 7 will join in if 4 invites her or 5 and 6 both invite her. Suppose the characteristic function v is defined as for every $S \subseteq N$,

$$v(S) = \begin{cases} 2 & \text{if } 7 \in S; \\ 1 & \text{if } \{1,2\} \cap S \neq \emptyset, 7 \notin S; \\ 0 & \text{otherwise.} \end{cases}$$

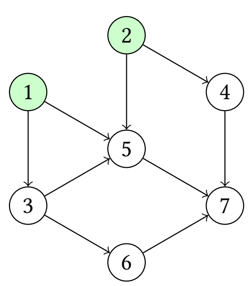

Figure 3: An example of a cooperative game in DAG. The green nodes are initial players.

5.1 Permission Structure with Mixed Approach

Note that there is no existing approach of permission structure that can handle the case in Example 5.1. Gilles et al. [4, 5] considered the cases where each player has to get permissions either from all or at least one of her superiors. Here we consider a more general case where each player can get permission from a partial subset of her superiors. A permission structure with mixed approach ϱ on N is a pair (P, Ψ) where P is a mapping $N \to 2^N$. The mapping P is asymmetric, i.e., for any pair $i, j \in N$, $j \in P(i)$ implies that $i \notin P(j)$ and j is called a superior of i. Define $P^{-1}(i) = \{j \in N \mid i \in P(j)\}$ as the set of i's successors. Notice that $P(i) = \emptyset$ if $i \in \mathcal{I}$. For a coalition $S \subseteq N$, the expression set L_S is recursively defined as

(1) $\xi_i \in L_S$ for any $i \in S$;
(2) $a \vee b \in L_S$ for any $a, b \in L_S$;
(3) $a \wedge b \in L_S$ for any $a, b \in L_S$.

Given an expression $\psi \in L_S$ and a coalition $T \subseteq N$, the evaluation $\psi(T)$ is the boolean result of ψ when $\xi_i = 1$ if $i \in T$ and $\xi_i = 0$ otherwise for all $i \in S$. Then the set $\Psi = \{\psi_i \in L_{P(i)} \mid i \in N\}$ consists of players' satisfiable expressions, where ψ_i indicates how her superiors hold the authority of permission: only when $\psi_i(T)$ is true, i can get the permission to create value in T. Specially, if $i \in \mathcal{I}$, ψ_i is always true since i does not need any others' permission. For instance, in Example 5.1, $\psi_5 = \xi_2 \vee (\xi_1 \wedge \xi_3)$ and $\psi_7 = \xi_4 \vee (\xi_5 \wedge \xi_6)$. With the generalized permission structure, an autonomous coalition now can be defined as follows.

Definition 5.2. A coalition $S \subseteq N$ is autonomous in the permission structure $\varrho = (P, \Psi)$ if for all $i \in S$, $\psi_i(S) = 1$.

Denote the set of all autonomous coalitions in ϱ by A_ϱ. We can observe several properties of A_ϱ as follows.

LEMMA 5.3. *Let ϱ be a permission structure on N, then (i) $\emptyset \in A_\varrho$, (ii) $N \in A_\varrho$ and (iii) for all $S, T \in A_\varrho$, $S \cup T \in A_\varrho$.*

PROOF. (i) Since there is no $i \in \emptyset$, then $\emptyset \in A_\varrho$. (ii) for all $\psi \in L_S$, $S \subseteq N$, $\psi(N) = 1$ since all variables are true. Thus, $N \in A_\varrho$. (iii) for all $i \in S \cup T$, if $i \in S$, $\psi_i(S) = 1$ implies $\psi_i(S \cup T) = 1$ since more variables get true; similarly for $i \in T$. Thus, $S \cup T \in A_\varrho$. □

Then we can define the largest autonomous part of a coalition.

Definition 5.4. Let ϱ be a permission structure on N. Then the largest autonomous part of a coalition $S \subseteq N$ is defined by

$$\alpha(S) = \bigcup \{T \mid T \subseteq S \text{ and } T \in A_\varrho\}.$$

Intuitively, $\alpha(S)$ is the largest autonomous sub-coalition of S, which suggests that for any player $i \in S \backslash \alpha(S)$, she cannot create value in coalition S. Similar to Section 3.2, we can map a characteristic function v to a projection v^ϱ on ϱ, where $v^\varrho(S) = v(\alpha(S))$, for every coalition $S \subseteq N$.

5.2 Weighted Shapley Value on Permission Structure

Now, we introduce weighted Shapley value with mixed permission structure as a class of mechanisms for diffusion cooperative game on DAGs.

Definition 5.5. For a cooperative game v on the player set N, given a permission structure ϱ and weights $\omega \in \mathbb{R}_+^{|N|}$, the weighted permission Shapley value with mixed approach for a player $i \in N$ is

$$\varphi_i^{\omega,\varrho}(v) = \varphi_i^{\omega}(v^\varrho).$$

As an example, if we apply the weighted permission Shapley value with mixed approach on the diffusion cooperative game in Example 5.1 and letting $\omega = 1^{|N|}$, then the reward distributed to each player is

$$\begin{cases} \varphi_1^{\omega,\varrho} = 11/15, \qquad \varphi_2^{\omega,\varrho} = 2/3, \\ \varphi_3^{\omega,\varrho} = \varphi_5^{\omega,\varrho} = \varphi_6^{\omega,\varrho} = \varphi_7^{\omega,\varrho} = 1/15, \\ \varphi_4^{\omega,\varrho} = 1/3. \end{cases}$$

Similar to the mechanisms in a forest, we can conclude that weighted Shapley value with mixed approach is a desirable mechanism that satisfies efficiency and diffusion incentive compatibility if the weight function is selected properly.

Definition 5.6. A weight function ω_i is proper if it only depends on d_i as $\omega_i = f(d_i)$, where d_i is the distance of player i to the initial players $\mathcal{I}$ in the graph, i.e. the minimum distance between i to one of the initial players ($\min_{j\in\mathcal{I}}\{d_{ji}\}$) and $f : \mathbb{N} \to \mathbb{R}_+$ is monotone non-decreasing.

Theorem 5.7. *For the monotone diffusion cooperative game in a DAG, if the reward distribution mechanism $\mathcal{M}$ is the weighted permission Shapley value with mixed approach with a proper weight function $\omega_i = f(d_i)$, then $\mathcal{M}$ is efficient and DIC.*

Proof. The efficiency can be easily derived since for all $\theta' \in \Theta$, $v^{\varrho}(J_{\mathcal{I}}(G(\theta'))) = v(J_{\mathcal{I}}(G(\theta')))$.

For the property of DIC, if we consider each player i and edge $e = (i, j) \in E$, there are two cases that may happen if i does not invite j given any possible report profile of others $\theta'_{-i} \in \Theta_{-i}$.

(i) if j cannot join the coalition, i.e., $j \notin J_{\mathcal{I}}(G(\theta'))$, then the proof is similar to that of Theorem 3.7 that shows player i will not get more reward without inviting j.

(ii) if j still can join the coalition, i.e., $j \in J_{\mathcal{I}}(G(\theta'))$, Let v^{ϱ} be the projection game when i invites j and $v^{\varrho'}$ be the projection game when i does not invite j. Suppose R is an order in $\mathcal{R}(N)$. Then we have

$$\begin{cases} C_i(v^{\varrho}, R) = C_i(v^{\varrho'}, R) & \text{if } i \text{ comes before } j \text{ in } R; \\ C_i(v^{\varrho}, R) \geq C_i(v^{\varrho'}, R) & \text{if } j \text{ comes before } i \text{ in } R. \end{cases}$$

The above (in)equalities hold because (1) if i is at the position before j, the marginal contribution of i is unchanged; (2) if i is at the position after j, she cannot bring j's contribution when she does not invite j. Note that d_k will not change for any player k with $d_k < d_j$. Let d'_j be the distance of player j to initial players if i does not invite j and hence $d'_j \geq d_j$. Thus, (1) if $d'_j = d_j$, then the weights of all players will not change and so do the weights of the orders, which can be computed from weights ω. Hence,

$$\varphi_i^{\omega,\varrho} = \sum_{R\in\mathcal{R}(N)} \omega_R C_i(v^{\varrho}, R) \geq \sum_{R\in\mathcal{R}(N)} \omega_R C_i(v^{\varrho'}, R) = \varphi_i^{\omega,\varrho'}$$

where $\varphi_i^{\omega,\varrho}$ is player i's reward when she invites j and $\varphi_i^{\omega,\varrho'}$ is player i's reward when she does not invite j. (2) if $d'_j > d_j$, then $f(d_j) \leq f(d'_j)$ since f is monotone non-decreasing. Let $R_{ij} \in \mathcal{R}(N)$ be some order where i comes before j and R_{ji} is the corresponding order where i and j's positions are exchanged in R_{ij}. We have $\frac{\omega_{R_{ji}}}{\omega_{R_{ij}}} \geq \frac{\omega'_{R_{ji}}}{\omega'_{R_{ij}}}$ (since R_{ij} is more likely sampled than R_{ji} with a larger $\omega(j)$). Hence, we have

$$\begin{aligned} \varphi_i^{\omega,\varrho} &= \sum_{R_{ij}\in\mathcal{R}(N)} \left[\omega_{R_{ij}} C_i(v^{\varrho}, R_{ij}) + \omega_{R_{ji}} C_i(v^{\varrho}, R_{ji})\right] \\ &\geq \sum_{R_{ij}\in\mathcal{R}(N)} \left[\omega'_{R_{ij}} C_i(v^{\varrho'}, R_{ij}) + \omega'_{R_{ji}} C_i(v^{\varrho'}, R_{ji})\right] \\ &= \varphi_i^{\omega,\varrho'}. \end{aligned}$$

The inequality in the second line holds since $\omega_{R_{ij}} + \omega_{R_{ji}} = \omega'_{R_{ij}} + \omega'_{R_{ji}}$, $C_i(v^{\varrho}, R_{ij}) \geq C_i(v^{\varrho'}, R_{ji})$ and $C_i(v^{\varrho}, R_{ij}) = C_i(v^{\varrho'}, R_{ij})$ (i.e., the larger term will obtain a larger factor). Therefore, in all cases i will not invite fewer agents. As a result, $\mathcal{M}$ is DIC. □

Finally, it is worth to point out that the weighted permission Shapley value that represents the solution of DARPA winning team also has a monotone non-decreasing weight function ω. Hence, it can be seen as **a diffusion incentive compatible extension in DAGs of DARPA winning team's solution**.

5.3 Applying on General Graphs

Finally, we discuss the possibility to apply our method on general networks. One may observe that in some real scenarios, the DAG we modelled is not necessarily the underlying social network. The underlying network could be any undirect graph. We have mainly focused on the DAG because in practice, a DAG could be the result of players' invitations associated with timestamp. Another reason to use DAG is that it is intuitive and handy to define permissions, which clearly specifies who permits/invites who.

In fact, our results can be extended to general undirected graphs. The only difficulty here is to define the permission structure which has many possible ways. We provide one feasible way to define the permission structure in a general undirected graph. First, define the permission set $P(i)$ of agent i to be all her neighbors via whom i can reach one of the sources with a simple path (this also works even if there are cycles), i.e., $P(i) = \{j \mid (i, j) \in E$ and there exists a simple path $i \to j \to \cdots \to s$ such that $s \in \mathcal{I}\}$. Then, i can get permissions from all, at least one, or a partial subset of agents in P_i as stated previously. Finally, it can be easily checked that all the results in this section still hold.

6 CONCLUSION

In this paper, we formalized the problem of diffusion incentives in cooperative games for the first time. We designed a family of reward distribution mechanisms such that the players are incentivized to invite their neighbours to join the coalition. The family of the reward distribution mechanisms combines the idea of the Shapley value with permission structure and weight system, which for the first time theoretically explained the winning solution of DARPA 2009 network challenge. We expect that our work will have a broad application on social networks such as crowdsourcing and question answering. One interesting future direction is to characterize the necessary and sufficient conditions for diffusion incentive compatibility.

REFERENCES

[1] Youngsub Chun. 1991. On the symmetric and weighted Shapley values. *International Journal of Game Theory* 20, 2 (1991), 183–190.

[2] Irinel C Dragan. 2008. *On the computation of Weighted Shapley Values for cooperative TU games*. Technical Report. University of Texas at Arlington.

[3] Theo SH Driessen. 2013. *Cooperative games, solutions and applications*. Vol. 3. Springer Science & Business Media.

[4] Robert P Gilles, Guillermo Owen, et al. 1999. *Cooperative games and disjunctive permission structures*. Tilburg University Tilburg, the Netherlands.

[5] Robert P Gilles, Guillermo Owen, and Rene van den Brink. 1992. Games with permission structures: the conjunctive approach. *International Journal of Game Theory* 20, 3 (1992), 277–293.

[6] Jin-Woo Jeong, Meredith Ringel Morris, Jaime Teevan, and Dan Liebling. 2013. A crowd-powered socially embedded search engine. In *Seventh International AAAI Conference on Weblogs and Social Media*.

[7] Ehud Kalai and Dov Samet. 1987. On weighted Shapley values. *International Journal of Game Theory* 16, 3 (1987), 205–222.

[8] Jon Kleinberg and Prabhakar Raghavan. 2005. Query incentive networks. In *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)*. IEEE, 132–141.

[9] Bin Li, Dong Hao, Hui Gao, and Dengji Zhao. 2022. Diffusion auction design. *Artificial Intelligence* 303 (2022), 103631.

[10] Bin Li, Dong Hao, Dengji Zhao, and Tao Zhou. 2017. Mechanism design in social networks. In *Thirty-First AAAI Conference on Artificial Intelligence*.

[11] Bezalel Peleg and Peter Sudhölter. 2007. *Introduction to the theory of cooperative games*. Vol. 34. Springer Science & Business Media.

[12] Galen Pickard, Wei Pan, Iyad Rahwan, Manuel Cebrian, Riley Crane, Anmol Madan, and Alex Pentland. 2011. Time-critical social mobilization. *Science* 334, 6055 (2011), 509–512.

[13] Tadeusz Radzik. 2012. A new look at the role of players' weights in the weighted Shapley value. *European Journal of Operational Research* 223, 2 (2012), 407–416.

[14] Lloyd S Shapley. 1953. A value for n-person games. *Contributions to the Theory of Games* 2, 28 (1953), 307–317.

[15] Adish Singla, Eric Horvitz, Pushmeet Kohli, Ryen White, and Andreas Krause. 2015. Information gathering in networks via active exploration. In *Twenty-Fourth International Joint Conference on Artificial Intelligence*.

[16] Arunesh Sinha and Michael P Wellman. 2019. Incentivizing Collaboration in a Competition. In *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems*. International Foundation for Autonomous Agents and Multiagent Systems, 556–564.

[17] René Van Den Brink. 1997. An axiomatization of the disjunctive permission value for games with a permission structure. *International Journal of Game Theory* 26, 1 (1997), 27–43.

[18] René van den Brink and Robert P Gilles. 1996. Axiomatizations of the conjunctive permission value for games with permission structures. *Games and Economic Behavior* 12, 1 (1996), 113–126.

[19] Eyal Winter. 2002. The shapley value. *Handbook of game theory with economic applications* 3 (2002), 2025–2054.

[20] H Peyton Young. 1985. Monotonic solutions of cooperative games. *International Journal of Game Theory* 14, 2 (1985), 65–72.

[21] Dengji Zhao. 2021. Mechanism Design Powered by Social Interactions. In *Proceedings of the 20th International Conference on Autonomous Agents and MultiAgent Systems*. 63–67.

[22] Dengji Zhao, Bin Li, Junping Xu, Dong Hao, and Nicholas R Jennings. 2018. Selling multiple items via social networks. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems*. International Foundation for Autonomous Agents and Multiagent Systems, 68–76.