On the Complexity of Controlling Amendment and Successive Winners

Extended Abstract

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ABSTRACT

Successive and amendment are two important sequential voting procedures widely used in parliamentary and legislative decision making. They have been extensively studied in the literature from different perspectives. However, investigating them through the lens of computational complexity theory has not been well conducted heretofore. This paper studies the parameterized complexity of constructive/destructive control by adding/deleting voters/candidates for these two procedures and provides a comprehensive parameterized complexity landscape of these problems.

KEYWORDS

amendment procedure; successive procedure; election control; parameterized complexity; NP-hard; W[1]-hard; W[2]-hard; FPT

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1 INTRODUCTION

Successive and amendment are two important sequential voting procedures widely used in practical parliamentary decision making [21]. They have been extensively and intensively investigated in the literature (see, e.g., [1, 7, 13, 15, 18, 21, 22]). Generally speaking, under these two procedures, we are given a set of candidates, a set of voters holding linear preferences over the candidates, and an agenda defined as a linear order over the candidates. The successive winner is the first candidate on the agenda such that there is a majority of voters who prefer this candidate to all successors of the candidate. The amendment procedure takes several rounds. Particularly, the winner of the first round is the first candidate on the agenda, and the i^{th} round's winner is either the i^{th} candidate on the agenda or the $(i-1)^{\text{th}}$ round's winner, determined by the head-to-head comparison between the two candidates with the winning one being the i^{th} round's winner. The amendment winner is the winner of the last round.

In practice, many factors can affect the outcome of an election. For instance, it is known that given the same voting profile, different agendas may result in different winners. Notably, by the characterizations of Black [7] and Miller [18], the successive procedure is more vulnerable to agenda control than the amendment procedure, in the sense that for the same profile, any candidate which can be made the amendment winner by some agenda can be also made the successive winner by some agenda (see [2] for an extension of the result). In order to provide a more fine-grained understanding of how election outcomes under the amendment and the successive procedures can be affected by different factors, Bredereck et al. [8] studied the complexity of several related combinatorial problems under these two procedures. Concretely, they studied Agenda Control, Coalition Manipulation, Possible Winner, Necessary Winner, and weighted variants of these problems. Their study shows that the amendment procedure is more resistant to agenda control and manipulation than the successive procedure.

To the best of our knowledge, the work of Bredereck et al. is so far the only one exploring the complexity of strategic problems under these two procedures, leaving many other types of strategic operations for these two procedures unexplored. Aiming at filling the gaps and significantly expanding our knowledge on to what extent the two procedures resist other types of strategic operations, we study several standard control problems. Particularly, we study the constructive control by adding/deleting voters/candidates problems first proposed in the pioneering paper of Bartholdi, Tovey, and Trick [3]. These problems model the scenario where a powerful election controller aims to make a distinguished candidate the winner by adding/deleting a limited number of voters/candidates. We also study the destructive counterparts of these problems first proposed by Hemaspaandra, Hemaspaandra, and Rothe [14]. Under the destructive control problems, the goal of the controller is to make the distinguished candidate not the winner. Investigating the complexity of these problems had partially dominated the early advance of computational social choice for quite a few years, ending up with an almost complete landscape of the complexity of these problems under many prominent voting procedures. We refer to [6, 12] for important progress by 2016 and refer to [10, 19, 24, 25, 27] for some recent results. It should be pointed out that the idea of election control has been adapted in many other settings like multiwinner voting [17, 26], judgement aggregation [4, 5], group identification [11, 28], tournament solutions [9], etc.

For the two procedures, we provide a comprehensive understanding of the complexity of election control problems, including many intractability results (NP-hardness, W[1]-hardness, W[2]-hardness), and numerous tractability results (P-results, FPT-results). Our main results are summarized in Table 1.

2 FORMAL DEFINITIONS

We assume the reader is familiar with the basics in (parameterized) complexity theory [20, 23].

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Table 1: A summary of the complexity of election control problems under the amendment and the successive procedures. Our results are in boldface. Results with the superscripts ★ and ∗ respectively mean that they hold when the distinguished candidate is the first and the last candidates on the agenda, with the superscript $\overline{\star}$ mean that they hold as long as the distinguished candidate is not the first one on the agenda, FPT-results with the superscripts ✠ and ¥ are respectively w.r.t. the number of predecessors and the number of successors of the distinguished candidate, and W[1]-hardness and W[2]-hardness results are w.r.t. the cardinality of the solution.

	CCAV-τ	CCDV-τ	DCAV-τ	DCDV-τ	CCAC-τ	CCDC-τ	DCAC-τ	DCDC-τ
Amendment	W[1]-h★ [16] **W[2]-h∗**	W[1]-h★ [16] **W[2]-h∗**	**FPT✠** **W[1]-h∗**	**FPT✠** **W[2]-h∗**	**P**	**P**	**P**	**P**
Successive	FPT✠ **W[1]-h∗**	FPT✠ **W[2]-h∗**	**P**	**P**	immune★ **W[2]-h$\overline{\star}$**	**W[1]-h★** **FPT¥**	**W[2]-h★** **FPT¥**	**FPT✠** **W[1]-h∗**

2.1 Elections

An election is a tuple (C, V) where C is a set of candidates, and V is a multiset of votes cast by a set of voters. Each $\succ \in V$ is a linear order over C, indicating the preference of the corresponding voter over C. For a candidate $a \in C$ and a subset $C' \subseteq C \setminus \{a\}$, we say that a beats C' w.r.t. V if there exists $V' \subseteq V$ s.t. $|V'| > |V|/2$, and for all $\succ \in V'$ and $c \in C'$ it holds that $a \succ c$. For uniformity, we define that every candidate $a \in C$ beats $\emptyset$. An agenda is a linear order $\rhd$ over C. We use $\rhd[i]$ to denote the i^{th} candidate in $\rhd$. Let $m = |C|$. For two integers i and j, we define $\rhd[i, j] = \{\rhd[x] : i \leq x \leq j\}$ if $1 \leq i \leq j \leq m$, and define $\rhd[i, j] = \emptyset$ otherwise. For each candidate $c \in C$, we call candidates before (resp. after) c on the agenda the predecessors (resp. successors) of c. The successive winner and the amendment winner w.r.t. (C, V) and $\rhd$ are determined as follows.

Successive Let $i \in [m]$ be the smallest integer s.t. $\rhd[i]$ beats $\rhd[i + 1, m]$ w.r.t. V. The successive winner is $\rhd[i]$.

Amendment This procedure takes several rounds. Precisely, the winner of the first round is $\rhd[1]$. For each $i = 2, 3, \ldots, m$, the winner of round i is determined as follows: letting $\rhd[j]$, $j \in [m]$, denote the winner of round $i - 1$, the winner of round i is $\rhd[j]$ if $\rhd[j]$ beats $\{\rhd[j + 1]\}$, and is $\rhd[j + 1]$ otherwise. The amendment winner is the winner of the last round.

2.2 Election Control Problems

Let τ be a voting procedure.

Constructive-Multimode-Control for τ (CMC-τ)	
Given:	A nonempty set C of registered candidates, a distinguished candidate $p \in C$, a set D of unregistered candidates, a multiset V of registered votes over $C \cup D$, a multiset W of unregistered votes over $C \cup D$, an agenda $\rhd$ over $C \cup D$, and four nonnegative integers k_{AV}, k_{DV}, k_{AC}, and k_{DC}.
Question:	$\exists C' \subseteq C \setminus \{p\}$, $D' \subseteq D$, $V' \subseteq V$, and $W' \subseteq W$ s.t. (1) $|C'| \leq k_{\text{DC}}$, $|D'| \leq k_{\text{AC}}$, $|V'| \leq k_{\text{DV}}$, $|W'| \leq k_{\text{AV}}$, (2) p is the τ winner of $((C \setminus C') \cup D', (V \setminus V') \uplus W')$ w.r.t. the agenda $\rhd$? [1]

Destructive-Multimode-Control for τ (DMC-τ) is defined similar to CMC-τ with only the difference that the goal is to make the given distinguished candidate not the winner. The eight standard control problems are special cases of CMC-τ and DMC-τ. The abbreviations of the names of the problems and their specifications are given in Table 2.

Table 2: Specifications of some election control problems. In the abbreviations, the letter X is either CC standing for "constructive control" or DC standing for "destructive control". For X=CC, the problems are special cases of CMC-τ, and for X=DC the problems are special cases of DMC-τ. The second letters A and D in the abbreviations respectively stand for "adding" and "deleting", and the third letters V and C respectively stand for "voters" and "candidates".

abbreviations	restrictions
XAV-τ	$k_{\text{AC}} = k_{\text{DC}} = k_{\text{DV}} = 0$ and $D = \emptyset$
XDV-τ	$k_{\text{AC}} = k_{\text{DC}} = k_{\text{AV}} = 0$ and $D = W = \emptyset$
XAC-τ	$k_{\text{DC}} = k_{\text{AV}} = k_{\text{DV}} = 0$ and $W = \emptyset$
XDC-τ	$k_{\text{AC}} = k_{\text{AV}} = k_{\text{DV}} = 0$ and $D = W = \emptyset$

3 CONCLUSION

We have investigated several election control problems under the amendment and the successive procedures, providing a comprehensive landscape of the complexity of these problems. Our results reveal that these two procedures behave quite differently regarding their resistance to these control types: the amendment procedure is more resistant to voter control types while the successive procedure is more resistant to candidate control types. Moreover, we reveal that the position of the distinguished candidate on the agenda has a significant impact on the parameterized complexity of the problems. Furthermore, we show that, from the complexity-theoretic perspective, most of these problems are more difficult to solve when the distinguished candidate p has a back position on the agenda than when p has a front position. We refer to Table 1 for a summary of our concrete results.

For future research, it is interesting to investigate if the P-results for destructive control problems can be extended to FPT-results for the corresponding resolute control problems [29] where there are multiple distinguished candidates whom the election controller would like to make nonwinners.

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