Representing and Reasoning about Auctions

JAAMAS Track

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ABSTRACT

In this paper, we propose a framework for representing and reasoning about auction-based protocols. Such a framework is of interest for building digital marketplaces based on auctions and should fulfill two requirements: (i) it should enable bidders to express their preferences over combinations of items and (ii) it should allow the mechanism designer to describe the rules governing the market, namely the legality of bids, the allocative choice, and the payment rule. To do so, we define a logical language in the spirit of the *Game Description Language*, namely *Auction Description Language* with a set of functions $\mathcal{F}_\mathcal{B}$ (ADL[$\mathcal{F}_\mathcal{B}$]). ADL[$\mathcal{F}_\mathcal{B}$] is expressive enough to represent different kinds of protocols and enables reasoning about auction properties, including playability, termination, and budget-balance. We also study the complexity of model-checking ADL[$\mathcal{F}_\mathcal{B}$].

KEYWORDS

Logics for Multi-agents; Game Description Language; Auction-based Markets

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1 INTRODUCTION

Auction-based markets are widely used for automated business transactions. There are numerous variants depending on the parameters considered, including the number of distinct items and their copies and the number of sellers and buyers [8, 9]. For a fixed set of parameters, the protocol, *i.e.*, the bidding, payment and allocation rules, may also differ. Building intelligent agents that can switch between different auctions and process their rules is a key issue for building automated auction-based marketplaces. In this setting, the auction designer should at first describe the rules governing the auction and second allow participants to express their preferences. The aim of this paper is to propose a language with clear semantics for enabling the representation of auctions as well as the reasoning about its rules and properties.

In the spirit of the *General Game Playing* (GGP) [5] where games are described with the help of *Game Description Language* (GDL), we previously introduced a logical language for describing auctions, denoted Auction Description Language (ADL) [11, 15]. We present ADL[$\mathcal{F}_\mathcal{B}$], which is described as *Auction Description Language* with a set of functions $\mathcal{F}_\mathcal{B}$. ADL[$\mathcal{F}_\mathcal{B}$] builds upon bidding languages, and hence provides a natural way to represent a wide range of protocols, ranging from single-units auctions to iterative combinatorial exchanges [17]. ADL[$\mathcal{F}_\mathcal{B}$] offers a unified perspective on an auction mechanism and it offers two benefits: (i) with this language, one can represent many kinds of auctions in a compact way and (ii) the precise state-transition semantics can be used to derive properties.

ADL[$\mathcal{F}_\mathcal{B}$] can be used for the automated verification of mechanism design properties and for automatically checking whether descriptions written in ADL[$\mathcal{F}_\mathcal{B}$] are well-formed. In this abstract, we illustrate the generality of ADL[$\mathcal{F}_\mathcal{B}$] by focusing on the simultaneous ascending auction (SAA) and we refer to the full paper for more examples based on combinatorial exchange and an evaluation of these protocols in terms of the aforementioned properties [12]. Finally, we show that when functions in $\mathcal{F}_\mathcal{B}$ can be computed in polynomial time, then the model-checking problem for a ADL[$\mathcal{F}_\mathcal{B}$]-formula is PTIME.

The use of GDL-based languages for describing market-based protocols have also been studied. De Jonge and Zhang [6, 7] discuss the use of GDL for modeling negotiation. The main advantage is being able to apply the existing domain-independent techniques from GGP. In another paper [4], they propose the use of GDL as a unifying language for defining general and complex negotiation domains. The closest contributions to ours is the Market Specification Language [18]. The main limitation is the single agent perspective and the lack of a clear link between the language, the mechanism formalization and the agents' preferences. Recent works have proposed using extensions of Strategy Logic (SL) [2, 16] for the verification and/or synthesis of auctions [1] and mechanism design [10, 13, 14]. SL is able to capture complex solution concepts such as Nash equilibrium but is computationally costly. In particular, model-checking SL formulas with perfect recall is NonElementary. This motivates the definition of languages that provide a reasonable cost-benefit for expressing and evaluating auctions.

2 AUCTION DESCRIPTION LANGUAGE

The Auction Description Language with a set of functions $\mathcal{F}_\mathcal{B}$ (ADL[$\mathcal{F}_\mathcal{B}$]) is a framework for the specification of auction-based markets. The logical language for ADL[$\mathcal{F}_\mathcal{B}$] is denoted by $\mathcal{L}_{\mathsf{ADL}[\mathcal{F}_\mathcal{B}]}$ and a formula φ in $\mathcal{L}_{\mathsf{ADL}[\mathcal{F}_\mathcal{B}]}$ is defined by the following grammar:

$$\varphi ::= p \mid initial \mid terminal \mid legal_{\mathrm{i}}(\beta) \mid does_{\mathrm{i}}(\beta) \mid \neg\varphi \mid \varphi \wedge \varphi \mid \bigcirc\varphi \mid z \leq z$$

where $p \in \Phi$ is a proposition, $\mathrm{i} \in \mathrm{N}$ is an agent, $\beta \in \mathcal{B}$ is an action and $z \in \mathcal{L}_z$ is a numerical term.

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(1) $initial \leftrightarrow price = \text{start} \wedge \bigwedge_{j\in G}\big(price_j = \text{start} \wedge \bigwedge_{i\in N}(\neg bid_{i,j} \wedge trade_{i,j} = 0)\big)$
(2) $sold_j \leftrightarrow \bigvee_{i\in N} trade_{i,j} = 1$, for each $j \in G$
(3) $terminal \leftrightarrow \neg initial \wedge \bigwedge_{j\in G}(sold_j \vee \bigwedge_{i\in N}\neg bid_{i,j})$
(4) $\bigcirc(trade_{i,j} = 1 \leftrightarrow bid_{i,j} \wedge \bigwedge_{r\in N\setminus\{i\}} \neg bid_{r,j})$, for each $i \in N$, $j \in G$
(5) $\bigcirc(trade_{i,j} = 0 \leftrightarrow \neg(bid_{i,j} \wedge \bigwedge_{r\in N\setminus\{i\}} \neg bid_{r,j}))$, for each $i \in N$, $j \in G$
(6) $legal_i(p_1, \ldots, p_m) \leftrightarrow \bigwedge_{j\in G}\big((p_j = 0 \wedge trade_{i,j} = 0) \vee (p_j = \text{sum}(price, \text{inc}) \wedge \neg sold_j) \vee (p_j = price_j \wedge trade_{i,j} = 1)\big)$, for each $i \in N$, $p_1, \ldots, p_m \in \{x : 0 \leq x < z_{\max} - \text{inc}\}$
(7) $\neg terminal \wedge price = x \rightarrow \bigcirc price = \text{sum}(x, \text{inc})$, for each $x \in I_{\geq 0}$
(8) $\neg terminal \wedge price_j = x \rightarrow \bigcirc((price_j = x \wedge sold_j) \vee (price_j = \text{sum}(x, \text{inc}) \wedge \neg sold_j))$, for each $j \in G$, $x \in I_{\geq 0}$
(9) $\bigcirc bid_{i,j} \leftrightarrow (does_i(p_1, \ldots, p_m) \wedge p_j \neq 0) \vee (bid_{i,j} \wedge terminal)$, for each $i \in N$, $j \in G$ and some $p_1, \ldots, p_m \in I_{\geq 0}$
(10) $payment_i = \text{sum}_{j\in G}(\text{times}(price_j, trade_{i,j}))$, for each $i \in N$

Figure 1: Simultaneous Ascending Auction with ADL$[\mathcal{F}_{\mathcal{B}}]$

Intuitively, *initial* and *terminal* specify the initial terminal states, resp.; $legal_i(\beta)$ asserts that agent i is allowed to take action β at the current state and $does_i(\beta)$ asserts that agent i takes action β at the current state. The formula $\bigcirc\varphi$ means "φ holds at the next state". The formula $z_1 \leq z_2$ means that the numerical term z_1 is smaller or equal to the numerical term z_2.

The semantics of ADL$[\mathcal{F}_{\mathcal{B}}]$ are based on state-transition models, which allows us to represent the key aspects of an auction, at first the legal bids and the transitions among states.

A state-transition-model (ST-*model* for short) M is a tuple $(W, \bar{w}, T, L, U, \pi_\Phi, \pi_Y)$, where: W is a nonempty set of *states*; $\bar{w} \in W$ is the *initial* state; $T \subseteq W$ is a set of *terminal* states; $L \subseteq W \times N \times \mathcal{B}$ is a *legality* relation, describing the legal actions at each state; $U : W \times \mathcal{B}^n \rightarrow W$ is an *update* function; $\pi_\Phi : W \rightarrow 2^\Phi$ is the valuation function for the state propositions; and $\pi_Y : W \times Y \rightarrow I$, is the valuation function for the numerical variables. A *path* δ inn M is a sequence of states and joint actions, representing a run or execution of an auction protocol.

The semantics for ADL$[\mathcal{F}_{\mathcal{B}}]$ is given in two steps[1]. First, we define a function $f_z : \mathcal{L}_z \times W \rightarrow I$ to compute the meaning of numerical terms $z \in \mathcal{L}_z$ in some specific state. Next, we define when a formula $\varphi \in \mathcal{L}_{\text{ADL}[\mathcal{F}_{\mathcal{B}}]}$ is true at the stage t of a path δ under a model M, denoted by $M, \delta, t \models \varphi$. The *model checking problem* for ADL$[\mathcal{F}_{\mathcal{B}}]$ is to determine whether $M, \delta, t \models \varphi$ or not.

Theorem 2.1. *Assuming that functions in $\mathcal{F}_{\mathcal{B}}$ can be computed in polynomial time, model checking* ADL$[\mathcal{F}_{\mathcal{B}}]$ *is in PTIME.*

3 EVALUATING PROTOCOLS

We can encode different properties of *direct revelation mechanisms* as ADL$[\mathcal{F}_{\mathcal{B}}]$-formulae (*e.g.*, individual rationality and budget balance). Proprieties such as strategyproofness and the constraints for well-formed descriptions (*e.g.*, termination and playability) can be inferred by meta-reasoning over the model specification.

For instance, a direct mechanism is strongly budget-balanced, SBB (resp. weakly budget-balanced, WBB) if the cumulative payments among all agents are exactly 0 (resp. non-negative). We denote the condition of a state being SBB by the following formula:

$$sbb =_{\text{def}} \text{sum}_{i\in N}(payment_i) = 0$$

The formula *wbb* is defined similarly, with ≥ instead of =.

Termination and playability are requirements for well-formed protocols. Termination means that each path from an ST-model reaches a terminal state, while playability means that there exists at least one action available in each reachable state.

4 REPRESENTING A SIMULTANEOUS ASCENDING AUCTION

Let us now illustrate how to encode a protocol using ADL$[\mathcal{F}_{\mathcal{B}}]$. We consider the simultaneous ascending auction (SAA), which is a single-side and single-unit auction similar to the traditional English auction, except that several goods are sold at the same time, and that the participants simultaneously bid for any number of goods they want [3]. Then, the rules of an SAA are formulated by ADL$[\mathcal{F}_{\mathcal{B}}]$-formulae as shown in Figure 1, where N is a set of agents, G is a set of good types, and $I_{\geq 0}$ is a set of non-negative integers.

In the initial state, no agent is bidding and the prices have the value start (Rule 1). A good is sold if it is traded to some agent (Rule 2). In a terminal state, all the goods are either sold or no one is bidding for them (Rule 3). A good will be traded to an agent in the next state if she is currently the only active bidder for this item, otherwise there is no trade (Rules 4-5). For each good, an agent can either bid the value 0, an increment on the current price (for unsold goods) or repeat her winning bid for this good (Rule 6). In a non-terminal state, the propositions and numerical variables are updated as follows: (i) the current price increases, (ii) the selling price increases for unsold goods, and (iii) the active bidders for each good are updated with respect to their bids (Rules 7-9). The payment for an agent is the cumulative value of the selling price for her traded goods (Rule 10). Let Σ_{sa} be the set of Rules 1-10.

Remark. The auction represented by Σ_{sa} is playable and terminates. Furthermore, it is WBB (but not SBB).

5 CONCLUSION

In this paper we presented ADL$[\mathcal{F}_{\mathcal{B}}]$, a unified framework for representing auction protocols. Our work is at the frontier of auction theory and knowledge representation. ADL$[\mathcal{F}_{\mathcal{B}}]$ provides tools for automated verification of properties from mechanism design. An ADL$[\mathcal{F}_{\mathcal{B}}]$ ST-model may represent direct mechanisms and be evaluated as such. Verifying a number of properties essentially comes down to model-checking ADL$[\mathcal{F}_{\mathcal{B}}]$-formulae, which can be done in PTIME when the functions in $\mathcal{F}_{\mathcal{B}}$ can be computed in polynomial time. Thus, ADL$[\mathcal{F}_{\mathcal{B}}]$ enables reasoning about important aspects of designing and playing auctions, while having a reasonable complexity cost.

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[1] The formal definition is available in the full paper [12].

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