Online 2-stage Stable Matching

Extended Abstract

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ABSTRACT

We focus on an online 2-stage problem, motivated by the following situation: consider a system where students shall be assigned to universities. There is a first stage where some students apply, and a first (stable) matching M_1 has to be computed. However, some students may decide to leave the system (change their plan, go to a foreign university, or to some institution not in the system). Then, in a second stage (after these deletions), we shall compute a second (final) stable matching M_2. As in many situations important changes to the assignments are undesirable, the goal is to minimize the number of divorces/modifications between the two stable matchings M_1 and M_2. Then, how should we choose M_1 and M_2? We show that there is an *optimal online* algorithm to solve this problem. In particular, thanks to a dominance property, we show that we can optimally compute M_1 without knowing the students that will leave the system. We generalize the result to some other possible modifications in the input (such as additional capacities of universities). We also tackle the case of more stages, showing that no competitive (online) algorithm can be achieved for the considered problem as soon as there are 3 stages.

KEYWORDS

Stable matching problem; on-line algorithm; 2-stage

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1 INTRODUCTION

Stable matchings have been extensively studied in the literature, both from a theoretical and a practical point of view. In the classical stable matching problem, one is given two equal-sized sets of agents, say men and women, where each person has strict preferences over the persons of the opposite sex. The goal is to match each man to exactly one woman and each woman to exactly one man, i.e., to find a perfect matching of men and women which is also *stable*. A perfect matching M is *stable* if there is no *blocking pair*, i.e., a pair of a man and a woman who are not matched together in M, but they prefer each other more to their current partners in the matching. In 1962, Gale and Shapley, in their seminal paper [11], showed that a stable matching always exists, and designed a polynomial-time algorithm that finds such a matching. The stable matching problem is motivated by various applications where a centralized automated matching scheme is necessary in order to assign positions to applicants (matching of interns to hospitals [22], [23], university admission [3], school placement [1], faculty recruitment [3], etc.). In most of these applications, the matching schemes employ extensions of the Gale and Shapley algorithm taking into account particular ingredients of each application, including the use of incomplete preference lists, the existence of ties, etc.

Given the dynamic nature of many applications, there is an increasing interest on matching-related problems in the setting of dynamic graph algorithms where vertices or edges arrive or leave over time. A first work in this direction was proposed by Khuler et al. [19] who considered the online stable marriage problem, where one is interested in the minimization of the number of blocking pairs. More recently, some studies are concerned with scenarios closely related to stable matchings, namely rank-maximal or (near) popular matchings [5], [15], [24]. Biro et al., in [6], studied the dynamics of stable marriage and stable roommates markets. Another interesting work in this setting is the one by Kanade, Leonardos and Magniez [18] who considered a setting where at each step, two random adjacent participants in some preference list are swapped and studied the problem of maintaining a matching while minimizing the number of blocking pairs.

A series of recent works tackle the situation where one wants to maintain stability of matchings when data evolves, while trying to minimize the modifications made in the matchings, as modifying pairs are usually highly non desirable in many applications:

- In [12], [13], [14], Genc et al. study the notion of robustness in stable matching problems by introducing (a, b)-supermatches. An (a, b)-supermatch is a stable matching such that: if a pairs break up, a new stable matching can be found by changing the partners of these a pairs and at most b other pairs. They also define the most robust stable matching as one that requires the minimum number of repairs (i.e., minimizes b) among all stable matchings.
- In [9], Chen et al. study the concepts of robustness where a matching must be stable even if the agents slightly change their preferences, and near stability where a matching must become stable if the agents slightly adjust their preferences.
- In [8], Bredereck et al. study a 2-stage incremental version of the stable matching problem in terms of parametererized complexity. More precisely, one is given: a preference profile $\mathcal{P}_1$ for stage one, a preference profile $\mathcal{P}_2$ for stage two, a stable matching M_1 for profile $\mathcal{P}_1$ and a nonegative integer k. The question is whether there is a stable matching for

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stage two, M_2, whose distance from M_1 is smaller than or equal to k. They also study the incremental version of the stable roommates problem. They perform a parameterized complexity analysis for both problems with respect to the "degree of change" both in the input (preference profiles) and the output (stable matchings).
- In [10], Gajulapalli et al. considered stable assignment in different settings of the school choice problem. As in the previous work, they consider a 2-stage problem, but here only the instance I_1 is known at stage one. The instance I_2 becomes available only at stage two. The authors consider different variants where given an optimal solution for the instance of the first stage, they seek a stable assignment of students to schools in two settings: In the first setting, it is disallowed to reassign the school of any student matched in stage one, and in the second setting the new stable assignment must provably minimize the number of such reassignments.

2 SUMMARY OF CONTRIBUTIONS

This work lies in this line of research, combining stability requirements and low number of modifications in dynamic stable matching problems. The main difference is that most of these works adopt a reoptimization-like framework [7], where *the first matching is fixed* and the question is how to modify it by respecting some given constraints. In our case, we consider a 2-stage situation and we want to compute in an online manner a pair of solutions, one for each of the two stages, minimizing the number of modifications between the first and the second stable matchings. Then, our main problem is how to choose the first stable matching without knowing the future so as to minimize the number of modifications in a 2-stage setting. Our approach is hence inspired by a new trend, the online multistage optimization framework [16], [4] and it is closely related to the 2-stage approach followed in [20] where a two-stage matching problem is considered in which the edges of the graph are revealed in two stages. Furthermore, we note that several admission procedures do use two-rounds (or multi-round procedures), for instance this is the case for national college admissions in Sweden, in Turkey, (previously) in France, or for high school admissions in New-York city (see [2], [17] and references therein).

Here, we focus on a 2-stage problem, motivated by the following situation: consider a system where students shall be assigned to universities. There is a first stage where some students apply, and a first (stable) matching M_1 has to be computed. However, some students may decide to leave the system (change their plan, go to a foreign university, or to some institution not in the system). Alternatively, some universities may decide to increase their capacities, if they receive a high number of demands. Then, in a second stage (after these deletions of students, or with these additional capacities), we shall compute a second stable matching M_2. The goal is to minimize the number of divorces/modifications between the two stable matchings M_1 and M_2. Then, how should we choose M_1 **and** M_2?

Let us formally define the problems we are looking at. We first consider the case without capacities (i.e., the classical stable matching framework). In the problem 2-LA-SMP (for 2-stage men-leaving women-arriving stable matching problem), we are given:
- Two sets U_1, U_2 of men, two sets W_1, W_2 of women, with $U_1 \supseteq U_2$ and $W_1 \subseteq W_2$.
- Each man in (resp. woman) gives his (her) preferences (total ranking) over the corresponding set of women (resp. men).

The goal is to compute two matchings (M_1, M_2) such that:
- M_1 is stable for (U_1, W_1) and M_2 is stable for (U_2, W_2).
- The number of divorces $|M_1 \setminus M_2|$ is minimized.

We are interested in the online version of the problem where we have to compute M_1 at stage 1 while having no knowledge about U_2, W_2. In other words, at stage 1, we only know U_1, W_1, and the preferences between men in U_1 and women in W_1. We note that these preferences between U_1 and W_1 do not change between the two stages. Our main result is the following theorem.

Theorem 2.1. *There is an optimal on-line algorithm for 2-LA-SMP.*

The main tool to prove this theorem is a dominance property, from which we deduce that choosing the men-optimal stable matching in the first stage is a dominant strategy. In other words, this is an optimal choice that we can make without knowing who will leave/enter the system in the second stage. Once this optimal choice is made in the first stage, the computation of M_2 boils down to solving a weighted stable matching problem, which can be done efficiently [21]. We notice that our optimal on-line algorithm is polynomial time.

We also show that this theorem generalizes to the more general college-admission case. This corresponds to the motivating example where, between stages one and two, some students may leave the system and some universities may have extra capacities. On the other hand, we note that when more modifications are allowed between the two stages (for instance both men and women may enter the system), then there does not exist optimal on-line algorithms anymore (and even no competitive on-line algorithms with constant ratios).

We finally tackle the case of more stages, showing that no competitive (online) algorithm exists for the considered problem as soon as there are 3 stages.

3 FUTURE WORKS.

We showed that the considered 2-stage stable matching problems admit an optimal online algorithm. While such an optimal online algorithm does not exist for more than 2 stages in the considered model, studying stable matching problems on more stages seems to be an interesting research direction. For instance, we can think of using randomized online algorithms to reach (asymptotic) competitive ratios, or make further assumptions on the model – for instance in several online matching problems people arrive one by one in the game. The study of the off-line problem could be also of interest, as well as extensions of the results to a more general preference model (with ties, incomplete preferences,…).

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