Minimally Constraining Line-of-Sight Connectivity Maintenance for Collision-free Multi-Robot Networks under Uncertainty

Extended Abstract

Yupeng Yang
University of North Carolina at Charlotte
Charlotte, United States
yyang52@uncc.edu

Yiwei Lyu
Carnegie Mellon University
Pittsburgh, United States
yiweilyu@andrew.cmu.edu

Wenhao Luo
University of North Carolina at Charlotte
Charlotte, United States
wenhao.luo@uncc.edu

ABSTRACT

In this paper, we consider the Line-of-Sight (LOS) connectivity maintenance under positional uncertainty for a team of robots consisting of multiple subgroups with given parallel tasks. The LOS connectivity between pairwise robots is preserved when the two robots are within the limited communication range and their LOS is occlusion-free from static obstacles over time. By unifying a control theoretic approach and a graph theoretic approach, we develop an Uncertainty Aware Line-of-Sight Minimum Spanning Tree (LOS-MST) framework to compute robots' motion that maintains only a minimally constraining set of LOS edges among robots for global and subgroup LOS connectivity, while minimizing the motion disruption to their original multi-robot behaviors. Simulation results are provided to validate the effectiveness of our proposed approach.

KEYWORDS

Line-of-Sight Communication; Connectivity Maintenance; Multi-Robot Networks; Robotics; Control Theory

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1 INTRODUCTION

Connectivity maintenance for multi-robot systems often concerns how to constrain robots' motion in order to keep the proximity-based multi-robot communication graph as one connected component, commonly referred to as maintaining *global connectivity* [2, 7, 9, 11–15]. For a team of robots consisting of multiple subgroups to perform parallel tasks [3, 4, 6, 7, 10], efficient local collaboration requires robots in the same subgroup to be connected as one component, and global connectivity across different subgroups is also needed for global situational awareness. In most of these works, however, two assumptions are commonly made such as (a) using disc-like communication models to describe the limited inter-robot communication capability, and (b) assuming accurate information about robots' locations for control design. These assumptions may not hold in many realistic environments, where Line-of-Sight (LOS) communication plays a critical role in modeling the performance of wireless signals amidst obstacles, e.g. mmWave and UAV communications. And robots could suffer from noisy observation due to sensor uncertainty in the real world.

Hence, in this paper we are interested in the problem of maintaining global and subgroup Line-of-sight connectivity for a team of mobile robots under noisy observation. Specifically, we formulate the minimally constrained LOS-aware coordination problem as a step-wise bi-level optimization problem, and propose the Uncertainty Aware Line-of-Sight Minimum Spanning Tree (LOS-MST) framework to *co-optimize* (a) the global and subgroup LOS connectivity topology to enforce, and (b) control deviation from nominal controllers subject to the LOS connectivity constraints. This allows robots to enjoy the greatest motion flexibility for their nominal task-related behaviors and respect the LOS connectivity constraints.

2 PROBLEM STATEMENT

We consider a robotic team $\mathcal{S}$ consisting of N robots moving in a d-dimension shared workspace, which consists of free space and occupied space $C_{\text{obs}} = \bigcup_{k=1}^{K} O_k$ by K static polyhedral obstacles $O_k \subset \mathbb{R}^d, \forall k$ whose positions are assumed to be known by the robots. The dynamics of each robot i located at the position $\mathbf{x}_i \in \mathbb{R}^d$ can be described as $\dot{\mathbf{x}}_i = \mathbf{u}_i$ with $\mathbf{u}_i \in \mathbb{R}^d$ as the control input. The Gaussian-distributed noisy observation available to the robots is denoted by $\hat{\mathbf{x}}_i = \mathbf{x}_i + \epsilon_i \in \mathbb{R}^d,\ \epsilon_i \sim \mathcal{N}(0, \sum_i)$. We assume the robotic team $\mathcal{S}$ has been assigned M simultaneous tasks ($M \leq N$) with M divided subgroups $\mathcal{S} = \{\mathcal{S}_1, \dots, \mathcal{S}_M\}$, where each robot i has been tasked to a subgroup $\mathcal{S}_m$ with the individual task-related nominal controller $\mathbf{u}_i = \hat{\mathbf{u}}_i \in \mathbb{R}^d$. Then given the real-time LOS communication graph $\mathcal{G}^{\text{los}}$ determined by the observed noisy robots' locations $\hat{\mathbf{x}} \in \mathbb{R}^{dN}$, the **objective** of this paper is to optimize the joint multi-robot controller $\mathbf{u} \in \mathbb{R}^{dN}$ such that (i) the global and subgroup LOS connectivity of the resulting LOS communication graph will be preserved as robots move, and (ii) the LOS constrained robot controller $\mathbf{u}$ for all the robots will be minimally deviated from their nominal task-related controller $\hat{\mathbf{u}} \in \mathbb{R}^{dN}$ so that the multi-robot task-related behaviors are minimally disrupted.

Global LOS Connectivity: A graph $\mathcal{G}^{\text{los}}$ is said to be *LOS connected* if there is at least one occlusion-free path between every pair of vertices on the graph with high probability.

Subgroup LOS Connectivity: A graph $\mathcal{G}^{\text{los}}$ is said to be *Subgroup LOS connected* if there is at least one occlusion-free path between every pair of vertices in each induced LOS subgroup graph $\mathcal{G}_m^{\text{los}} = \mathcal{G}^{\text{los}}[\mathcal{V}_m] \subseteq \mathcal{G}^{\text{los}}, \forall m = 1, \dots, M$ with high probability, where $\mathcal{V}_m \subseteq \mathcal{V}$ contains all robots within the same sub-group $\mathcal{S}_m$.

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Note that any edge $(v_i, v_j) \in \mathcal{E}^{\text{los}}$ is said to exist in a LOS communication graph $\mathcal{G}^{\text{los}}$ when it satisfies (a) communication distance constraint $||\mathbf{x}_i - \mathbf{x}_j|| \leq R_{\text{c}}$ with R_{c} as the limited communication range, and (b) occlusion-free condition with $\mathbf{x}_i(1-\beta) + \mathbf{x}_j\beta \notin \mathcal{C}_{\text{obs}}, \ \forall \beta \in [0,1]$. We assume the global and subgroup connectivity of the LOS communication graph $\mathcal{G}^{\text{los}}$ are satisfied initially. Hence, the step-wise optimization problem can be defined as follows, boiling down to (a) find the least constraining LOS subgraph $\mathcal{G}^{\text{slos}}$ to maintain, and (b) minimize the control deviation subject to the LOS constraints computed from (a).

$$\mathbf{u}^* = \underset{\mathcal{G}^{\text{slos}},\mathbf{u}}{\arg\min} \sum_{i=1}^{N} ||\mathbf{u}_i - \hat{\mathbf{u}}_i||^2 \tag{1}$$

$$\text{s.t.} \quad \mathcal{G}^{\text{slos}} = (\mathcal{V}, \mathcal{E}^{\text{slos}}) \subseteq \mathcal{G}^{\text{los}} \quad \text{is LOS connected} \tag{2}$$

$$\mathcal{G}^{\text{slos}}_m = \mathcal{G}^{\text{slos}}[\mathcal{V}_m] \quad \text{is LOS connected}, \quad \forall m = 1, \ldots, M \tag{3}$$

$$\mathbf{u} \in S_{\mathbf{u}}^{\sigma^s} \bigcap S_{\mathbf{u}}^{\sigma^{\text{sobs}}} \bigcap C_{\mathbf{u}}^{\sigma^c}(\mathcal{G}^{\text{slos}}) \bigcap \mathscr{C}_{\mathbf{u}}^{\sigma^{\text{los}}}(\mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}}), \tag{4}$$

$$||\mathbf{u}_i|| \leq u_{\max}, \forall i = 1, \ldots, N$$

where $S_{\mathbf{u}}^{\sigma^s}, S_{\mathbf{u}}^{\sigma^{\text{sobs}}}, C_{\mathbf{u}}^{\sigma^c}(\mathcal{G}^{\text{slos}}), \mathscr{C}_{\mathbf{u}}^{\sigma^{\text{los}}}(\mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}})$ represent respectively the corresponding admissible control space for high-probability inter-robot collision avoidance ($S_{\mathbf{u}}^{\sigma^s}$), robot-obstacle collision avoidance ($S_{\mathbf{u}}^{\sigma^{\text{sobs}}}$), communication distance constraints ($C_{\mathbf{u}}^{\sigma^c}(\mathcal{G}^{\text{slos}})$), and occlusion-free constraint ($\mathscr{C}_{\mathbf{u}}^{\sigma^{\text{los}}}(\mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}})$). $u_{\max}$ indicates the bounded control input for each robot.

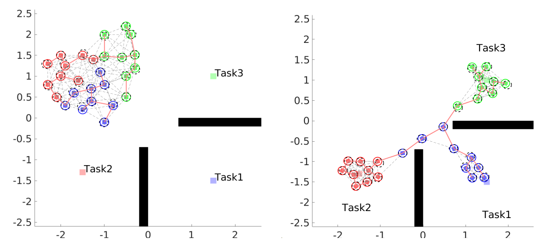
3 METHOD

In [5], we introduced Probabilistic Safety Barrier Certificates (PrSBC), a probabilistic extension based on Control Barrier Function (CBF) [1] to ensure high-probability collision avoidance of each pairwise robots and robot-obstacle accounting for uncertainties [8]. The PrSBC has been presented in the form of deterministic linear control constraints to characterize the admissible control space $S_{\mathbf{u}}^{\sigma^s}, S_{\mathbf{u}}^{\sigma^{\text{sobs}}}$ for guaranteed safety with probability at least $\sigma^s, \sigma^{\text{sobs}} \in (0,1)$. Likewise, one can also adopt the same technique to describe the deterministic admissible control space $C_{\mathbf{u}}^{\sigma^c}(\mathcal{G}^{\text{slos}})$, so that with probability at least $\sigma^{\text{c}} \in (0,1)$, all pairwise communication distance constraints will be satisfied between robots i, j in a given spanning LOS subgraph $\mathcal{G}^{\text{slos}} = (\mathcal{V}, \mathcal{E}^{\text{slos}})$ where $(v_i, v_j) \in \mathcal{E}^{\text{slos}}$.

To address the occlusion-free condition, we propose to use a Minimum Volume Covering Ellipsoid (MVCE) [16] to circumscribe (i) the corresponding LOS edge and (ii) the σ^{los}-confidence error ellipsoids $\mathcal{Q}_i^{\sigma^{\text{los}}}, \mathcal{Q}_j^{\sigma^{\text{los}}}$ defined by the Gaussian distributions of the noisy positions of the robots i, j. This enables us to construct the novel Probabilistic Line-of-Sight Connectivity Barrier Certificates (PrLOS-CBC) as the admissible control space $\mathscr{C}_{\mathbf{u}}^{\sigma^{\text{los}}}(\mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}})$, from which the resulting LOS edges of $\mathcal{G}^{\text{slos}}$ between moving robots will stay occlusion-free from the obstacles with a probability at least $\sigma^{\text{los}} \in (0,1)$. Thus by following the admissible control constraints depicted as $S_{\mathbf{u}}^{\sigma^s}, S_{\mathbf{u}}^{\sigma^{\text{sobs}}}, C_{\mathbf{u}}^{\sigma^c}(\mathcal{G}^{\text{slos}}), \mathscr{C}_{\mathbf{u}}^{\sigma^{\text{los}}}(\mathcal{C}_{\text{obs}}, \mathcal{G}^{\text{slos}})$, the moving robots are able to ensure collision-free motion while preserving the required LOS connectivity through a given subgraph $\mathcal{G}^{\text{slos}}$ to maintain, all with prescribed high probabilities.

At each time step, there may exist multiple subgraphs $\mathcal{G}^{\text{slos}}$ that satisfy global and subgroup LOS connectivity. In order to determine the optimal subgraph $\mathcal{G}^{\text{slos}*}$ to maintain, we propose the Uncertainty-Aware Line-of-Sight Minimum Spanning Tree (LOS-MST). By taking as inputs the robots' noisy positions and nominal controllers, the algorithm is able to find a particular minimum spanning tree $\mathcal{G}^{\text{slos}*} \subseteq \mathcal{G}^{\text{los}}$ at each time step, whose edges 1) are least likely to break under the nominal multi-robot behaviors, and 2) satisfy the global and subgroup LOS connectivity requirement. Then by maintaining such a subgraph $\mathcal{G}^{\text{slos}} = \mathcal{G}^{\text{slos}*}$ with the associated admissible control space in (4), the problem (1) becomes a Quadratic Programming that could be efficiently solved in real-time.

4 RESULTS



(a) t = 0 **(b) t=2000**

Figure 1: Simulation example of $N = 27$ robots moving to three different places as the assigned parallel tasks.

As shown in Figure 1, a team of $N = 27$ mobile robots has been assigned individual nominal controllers for them to move toward three different task sites. The grey dashed edges represent the current LOS edges from $\mathcal{G}^{los}$ among robots, and the red edges are those from the computed optimal subgraph $\mathcal{G}^{slos*} \subseteq \mathcal{G}^{los}$ to actively maintain at each time step. The black rectangles represent the static obstacles. Every robot only has access to the Gaussian-distributed noisy observations of their positions marked by a dashed black circle covering each robot. As shown in the figure, our proposed method can ensure global and subgroup LOS connectivity over time by overwriting nominal controllers as necessary, while allowing robots to stay as close to their designated task site.

5 CONCLUSION

In this paper, we present a bilevel optimization-based control framework for constrained multi-robot coordination with global and subgroup Line-of-Sight connectivity maintenance. In contrast to most existing work that assumes perfect location information about robots, we relax the assumption and consider the Gaussian-distributed positional noise. By employing a set of CBF-based probabilistic barrier certificates to enforce collision-free behaviors and LOS edge connectivity with high probability under uncertainty, we propose the Uncertainty-Aware Line-of-Sight Minimum Spanning Tree (LOS-MST) approach that enables robots to identify and maintain a dynamic set of critical LOS edges for required connectivity maintenance, while introducing the least motion disruptions for them to execute the original parallel tasks. Simulation results are provided to demonstrate the performance our proposed algorithm.

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