Altruism in Facility Location Problems

Extended Abstract

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ABSTRACT

We study the facility location problems (FLPs) with altruistic agents who act to benefit others in their affiliated groups. Our aim is to design mechanisms that elicit true locations from the agents in different overlapping groups and locate a facility to serve agents to approximately optimize a given objective based on agents' costs to the facility. Existing studies of FLPs consider *myopic* agents who aim to minimize their own costs to the facility, while we mainly consider *altruistic* agents who consider the group costs incurred by all agents in their groups. Accordingly, we define Pareto strategyproofness to account for this new type of agents and their multiple group memberships with incomparable group costs. We consider mechanisms satisfying this strategyproofness under various combinations of the planner's objectives and agents' group costs. For each of these settings, we provide upper and lower bounds of approximation ratios of the mechanisms satisfying the Pareto strategyproofness.

KEYWORDS

Facility Location; Mechanism Design; Altruism

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1 INTRODUCTION

In recent decades, facility location problems (FLPs) have been widely studied within the context of mechanism design without money [2]. In the most basic mechanism design version of FLPs, initiated and studied by Moulin [12] and Procaccia and Tennenholtz [14], a planner seeks to locate a facility (e.g., school, library, or park) to best serve a set of agents in a region based on the ideal locations of the agents. Because agent ideal locations are unknown to the planner, the planner must elicit agent locations in order to determine a facility location that best serves the agents. As there is a potential for the agents to misreport private locations to manipulate the facility location, the main research agenda in mechanism design for FLPs is to design a strategyproof mechanism to elicit the true private agents' locations and locate the facility that (approximately) optimizes a given planner's objective (e.g., minimizing the total or maximum distance of the agents to the facility).

Previous mechanism design studies in FLPs have focused only on *myopic* (e.g., selfish) agents in which each agent cares about their own cost to the facility (e.g., the distance between their ideal location and the facility location). However, in many real-world settings (e.g., group decision-making), agents exhibit group behavior [8], altruistic behavior [13, 16], or prosocial behavior [6, 15], where altruistic agents act to benefit others in their affiliated groups without expecting anything in return. Motivated by the altruistic behavior of agents in real-world situations, our focus is to study and model altruistic agents in FLPs. Conceptually, FLPs with altruistic agents model situations ranging from altruistic agents advocating for facility accessibility of their own groups to altruistic agents lobbying for a committee to represent their group views on some issue. For instance, when analyzing voter behavior, various theories (e.g., the altruism theory of voting) have studied the social preferences of voters that consider the welfare of others [5, 9]. In order to model these situations with altruistic agents, we address the following key questions.

(1) How should one model altruistic agents in FLPs?
(2) How should one design desirable mechanisms to (approximately) optimize a given objective with altruistic agents?

When modeling altruistic agents in FLPs, an altruistic agent should be able to express concerns over their group members and consider the group costs incurred by all agents in their groups rather than their own cost to the facility (e.g., the distance of other agents to the facility or the welfare of the other agents). As a result, the altruistic agents' costs and their strategic decisions of misreporting information can be affected by their concerns toward groups, which ultimately impact the facility location and mechanisms' desirable properties. In order to study FLPs with altruistic agents, we (1) propose various (group) costs for the altruistic agents based on their affiliated groups, (2) introduce definitions for altruistic agents to be truthful (when an agent belongs to multiple groups with incomparable group costs), and (3) design mechanisms that satisfy the proposed truthfulness definitions and approximately optimize several objectives.

1.1 Related Work

The classical (mechanism design variants of) facility location problems (FLPs) were first studied by Moulin [12], which characterized mechanisms that are strategyproof, Pareto efficient, and anonymous for single-peaked preferences on a line. However, finding strategyproof mechanisms with good approximation ratios in FLPs remains a challenging problem, which was first studied by Procaccia and Tennenholtz [14]. They proved that putting the facility at the median and the leftmost location can achieve tight approximation ratios while guaranteeing the strategyproofness for minimizing the social cost and the maximum cost, respectively. However, we observe that neither of the mechanisms we mentioned above

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can guarantee Pareto strategyproofness when agents are altruistic. Other works and variations on FLPs can be found in a recent survey [2].

One of the important notions in our paper is group fairness. Recently, there is an increased interest in studying fairness in FLPs [1, 3, 4, 10, 11, 17]. Cai et al. [1] and Chen et al. [3] studied the minimax envy objective that aims to minimize the (normalized) maximum difference between any two agents' costs. In addition, Ding et al. [4] and Liu et al. [11] studied the envy ratio objective, which aims to minimize the maximum over the ratios between any two agents' utilities, and Lam [10] considered the Nash Welfare objective in FLPs. All of these works considered fairness for individual agents, and there is no notion of group memberships. The work of Filos-Ratsikas and Voudouris [7] studied the FLPs with agents who are partitioned into multiple districts with equal size, which can be regarded as each agent having her own group. They focused on the social cost objective, rather than group-fair objectives. Here we emphasize the work [17], which studied group-fair FLPs with (disjoint) groups (i.e., each agent belongs to a single group) and group-fair objectives (including *mtgc* and *magc*) for myopic agents. In our paper, we also study two group-fair objectives they proposed for altruistic agents to complete the picture of group fairness in FLPs.

2 PROBLEM STATEMENT

Let $N = \{1, 2, ..., n\}$ be a set of agents on the real line and $G = \{G_1, ..., G_m\}$ be the set of groups of agents. Each agent $i \in N$ has profile $r_i = \{x_i, g_i\}$ where $x_i \in \mathbb{R}$ is the location reported by agent i and $g_i \subseteq \{1, ..., m\}$ is the group membership of agent i. We use $|G_j|$ to denote the number of agents in group G_j and $|g_i|$ to denote the number of groups agent i belongs to. Without loss of generality, we assume that $x_1 \leq x_2 \leq ... \leq x_n$. A profile $r = \{r_1, r_2, .., r_n\}$ is a collection of locations and group memberships of agents. A deterministic mechanism is a function f which maps profile r to a facility location $y \in \mathbb{R}$. Let $d(a, b) = |a - b|$ be the distance between any $a, b \in \mathbb{R}$.

We consider the two classical cost objectives, minimizing the social cost and the maximum cost [14]. Given a facility location y and true profile r, the social cost and the maximum cost are defined as $sc(y, r) = \sum_{i \in N} d(y, x_i)$, and $mc(y, r) = \max_{i \in N} \{d(y, x_i)\}$. Besides the classical cost objectives, we consider two group-fair cost objectives [17]. One is minimizing the maximum total group cost (*mtgc*), $mtgc(y, r) = \max_{1 \leq j \leq m} \left\{\sum_{i \in G_j} d(y, x_i)\right\}$. The other is minimizing the maximum average group cost (*magc*), which is defined as $magc(y, r) = \max_{1 \leq j \leq m} \left\{\sum_{i \in G_j} d(y, x_i)/|G_j|\right\}$.

Our goal is to design mechanisms that enforce Pareto strategyproofness (discussed below) while approximately optimizing an objective function when the agents are altruistic. We measure the performance of a mechanism f by comparing the objective that f achieves and the objective achieved by the optimal solution. If there exists a number α such that for any profile r, the output from f is within α times the objective achieved by the optimal solution, then we say the approximation ratio of f is α.

We focus our efforts on *altruistic cost*, which we introduce and define in two ways, the altruistic total cost and the altruistic maximum cost. For all agents in group G_j, the altruistic total cost is the total cost of the agents in group G_j, $uc(y, G_j) = \sum_{i \in G_j} d(y, x_i)$, and the altruistic maximum cost is the maximum cost among the agents in G_j, $uc(y, G_j) = \max_{i \in G_j} \{d(y, x_i)\}$. If an agent is in multiple groups, the agent has multiple altruistic costs (one per group), thus we introduce Pareto strategyproofness.

Objectives	Cost Functions	
	Altruistic total cost	Altruistic max cost
social cost	UB: $2m - 1$ LB: m	UB: $\max\{\frac{n}{2}, 1\}$ LB: $\max\{\frac{n}{2}, 1\}$
maximum cost	UB: 2 LB: 2	UB: 2 LB: 2
mtgc†	UB: 3 LB: 2	UB: $\max\{\frac{\lvert G_{max}\rvert}{2}+1, 2\}$ LB: $\max\{\frac{\lvert G_{max}\rvert}{2}, 2\}$
magc†	UB: 3 LB: 2	UB: $\max\{\frac{\lvert G_{max}\rvert}{2}+1, 2\}$ LB: $\max\{\frac{\lvert G_{max}\rvert}{2}, 2\}$

Table 1: Result Summary. UB: upper bound. LB: lower bound. $|G_{min}|$ ($|G_{max}|$) is the size of the smallest (largest) group. †For $m = 1$, *mtgc* and *magc* are equivalent to the social cost, thus the results of the social cost hold for them.

Definition 1. *A mechanism f is* ***Pareto strategyproof (PSP)*** *if and only if an agent cannot benefit at least one of her groups without hurting any other group she belongs to by reporting a false location. More formally, given any profile r, let $r'_i = \{x'_i, g_i\}$ be a profile with the false location reported by agent i. For agent i, we have $\exists j \in g_i$, $uc(f(r), G_j) < uc(f(r'_i, r_{-i}), G_j)$ or $\forall j \in g_i$, $uc(f(r), G_j) = uc(f(r'_i, r_{-i}), G_j)$ where r_{-i} is the profile of all agents except agent i.*

2.1 Our Contribution

In our work, we have two main points of contribution, one conceptual and one technical.

Conceptual Contribution. Different from the previous work which only considers the myopic agents whose costs are their own distances from the facility, we study the altruistic agents who care about their groups and define the altruistic cost which depends on their group memberships. In addition, as agents can belong to multiple groups and the altruistic costs across different groups are incomparable, we propose *Pareto strategyproofness (PSP)* concept to ensure that each agent cannot misreport their location to benefit their groups simultaneously.

Technical Contribution. Table 1 summarizes our results. For the altruistic total cost, we show that the majority group median mechanism proposed by Zhou et al. [17], which puts the facility at the median of the largest group, is PSP and give the respective upper and lower bounds for the classic objectives. We also propose a new PSP mechanism for minimizing the classical objectives which has a better approximation ratio when each agent cannot belong to many groups at the same time. For the group-fair objectives, we reuse the majority group median mechanism and show that the results of the approximation ratios in Zhou et al. [17] can be adapted here. For the altruistic maximum cost, we observe that none of the mechanisms we mentioned above is PSP. We design new PSP mechanisms in this setting. We first propose a mechanism which achieves tight bounds for both the social cost and the maximum cost. For the group-fair objectives, we also propose a new mechanism and provide upper and lower bounds for each objective.

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